

Binomial

Introduction

In algebra, a binomial is an expression that has two unlike terms connected through an addition or subtraction operator in between. For example, $2xy + 7y$ is a binomial since there are two terms. Algebraic expressions can be categorized into different types depending upon the number of terms present, like monomial, binomial, trinomial, etc. In this article, we will explore the binomial expression in algebra, its properties and its identities that are used to solve various problems in algebra. We shall go through different solved examples based on binomial for a better understanding of the concept.

What is a Binomial?

A binomial is an algebraic expression that consists of exactly two terms separated by addition (+) or subtraction (-). These terms can include variables, coefficients, and constants. Binomials are foundational in algebra and have widespread applications in mathematics, science, and engineering.

Binomial Meaning

The word binomial comes from the Latin words "bi" meaning two and "nomial" meaning term. In algebra, a binomial is a polynomial with exactly two terms. Some common examples of binomials include: $x + 5$, $3y - 7$, $2a^2 + 4b$, $m^3 - 9n$, etc.

Each of these expressions contains two distinct terms, which makes them binomials.

Addition of Binomials

Addition of two binomials is done only when it contains like terms. This means that it should have the same variable and the same exponent.

Example:

Let us consider, two binomials

$$10x^3 + 4y \text{ and } 9x^3 + 6y$$

$$\text{Adding both the equation} = (10x^3 + 4y) + (9x^3 + 6y)$$

$$\text{Therefore, the resultant equation} = 19x^3 + 10y$$

Subtraction of Binomials

Subtraction of two binomials is similar to the addition operation as if and only if it contains like terms.

Example:

$$12x^3 + 4y \text{ and } 9x^3 + 10y$$

Subtracting the above polynomials, we get:

$$(12x^3 + 4y) - (9x^3 + 10y)$$

$$= 12x^3 + 4y - 9x^3 - 10y$$

$$\text{Therefore, the resultant equation is} = 3x^3 - 6y$$

Multiplication of Binomials

When multiplying two binomials, the distributive property is used. It is generally referred to as the FOIL method. Because in this method, multiplication is carried out by multiplying each term of the first factor to the second factor. So, in the end, multiplication of two two-term polynomials is expressed as a trinomial.

For example, $(mx + n)(ax + b)$ can be expressed as $max^2 + (mb + an)x + nb$

Factorising Binomials

Factorising binomials involves rewriting them as the product of two or more factors. Various methods exist for factoring binomials:

1. Finding the Greatest Common Factor (GCF)

The greatest common factor (GCF) is the largest factor shared by both terms.

Example: Factor $6x + 9$

The GCF is 3, so the factored form is $3(2x + 3)$.

2. Factorising the Difference of Squares

A binomial in the form $a^2 - b^2$ follows the rule:

Example: Factorise $x^2 - 16$

$$x^2 - 16 = (x - 4)(x + 4)$$

3. Factorising the Sum or Difference of Cubes

A binomial in the form $a^3 + b^3$ or $a^3 - b^3$ follows these rules:

- $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

- $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Example: Factor $x^3 - 8$

$$x^3 - 8 = (x - 2)(x^2 + 2x + 4)$$

Squaring a Binomial

Squaring a binomial means multiplying it by itself:

- $(a + b)^2 = a^2 + 2ab + b^2$

- $(a - b)^2 = a^2 - 2ab + b^2$

Examples:

- $(x + 3)^2 = x^2 + 6x + 9$

- $(2y - 5)^2 = 4y^2 - 20y + 25$

Important Notes on Binomials

- A binomial always consists of two terms.
- Factoring binomials helps in simplifying expressions and solving equations.
- Squaring a binomial follows specific formulas.
- Binomials have practical applications in probability, finance, physics, and engineering.

Real-life applications of binomials

- **Algebraic Calculations and Expansions:** Binomial expressions are used to simplify complex algebraic equations and solve real-world mathematical problems.

- **Physics (Motion and Forces):** Used in kinematics equations involving two different parameters like velocity and acceleration.

Conclusion

Binomials are an essential part of algebra and mathematics in general. Understanding their structure, factoring methods, and binomial expansion is crucial for solving complex algebraic problems. Whether working with simple algebraic expressions or advanced polynomial expansions, mastering binomials helps in problem-solving across various mathematical disciplines.

Frequently Asked Questions (FAQ)

1. What is a binomial in simple terms?

A binomial is an algebraic expression that consists of exactly two unlike terms, such as $x + 4$ or $3y - 5$.

2. How do you factor a binomial?

Factoring a binomial depends on its form. Common techniques include finding the greatest common factor, using the difference of squares, and recognizing sum and difference of cubes.

3. How do you square a binomial?

To square a binomial, use the formulas:

- $(a + b)^2 = a^2 + 2ab + b^2$
- $(a - b)^2 = a^2 - 2ab + b^2$

4. How do you cube a binomial?

- $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
- $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

5. What are some real-life applications of binomials?

Binomial expressions are used to simplify complex algebraic equations and solve real-world mathematical problems.