

Tan 90°: Definition, Value & Real-World Applications

Discover the value, derivation, and real-world applications of Tan 90° in physics, engineering, and daily life. Learn how to solve problems step-by-step.

Introduction

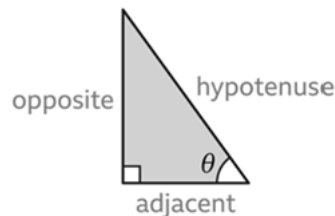
Imagine a rocket taking off from the ground vertically. The angle between the rocket and the ground approaches to 90° as it rises. In trigonometry and scientific applications, it is essential to understand the tangent function at this angle.

In this article, we'll explore:

- What is Tan 90°?
- What is the value of Tan 90°?
- Where is Tan 90° applied in real life?

Tangent

Tangent, written as tan, is a fundamental trigonometric function that represents the ratio of the opposite side to the adjacent side in a right-angled triangle. Mathematically, for any angle θ in a right triangle:



$$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\text{Opposite side}}{\text{Adjacent side}}$$

What is Tan 90°?

The value of the tangent trigonometric function for an angle of 90° between the opposite side and the adjacent side is known as Tangent 90 degree. The formal notation for the tangent of a ninety-degree angle is $\tan 90^\circ$.

Value of Tan 90°

The value of $\tan 90^\circ$ is undefined. This is because, in a right-angled triangle, when the angle reaches 90°, the adjacent side becomes zero. Since division by zero is not defined, $\tan 90^\circ$ is considered undefined.

$$\tan 90^\circ = \frac{\text{Opposite side}}{0} = \text{Undefined}$$

Thus, $\tan 90^\circ$ does not have a finite value.

Value of Tan 90° in radians

We know that

$$\theta \text{ in radians} = \frac{\pi}{180^\circ} \times \theta \text{ in degrees}$$

$$= 90^\circ \times \left(\frac{\pi}{180^\circ}\right)$$

$$= \frac{1}{2} \times \pi = \frac{\pi}{2}$$

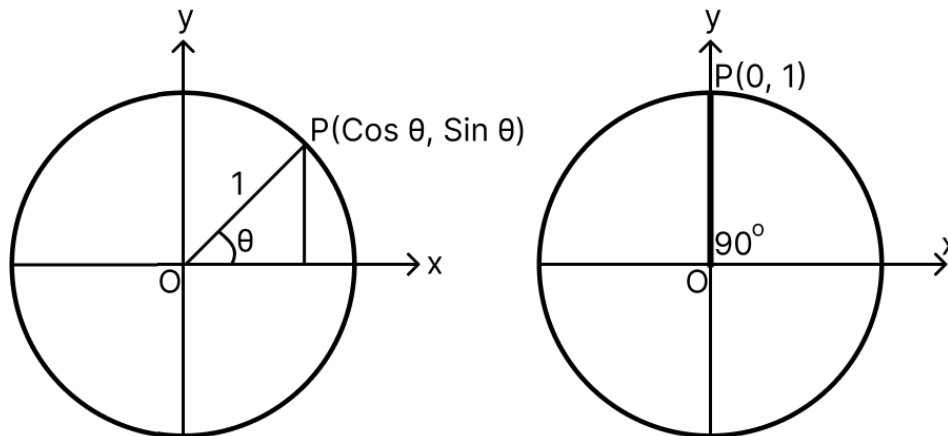
$$\text{Tan } 90^\circ = \text{Tan } \frac{\pi}{2} = \text{not defined}$$

Value of Tan 90° using a unit circle

On a unit circle, the hypotenuse is always 1 since the radius of the unit circle is 1. We know that

$$\text{Cos } \theta = \frac{\text{Adjacent side}}{\text{hypotenuse}}$$

Since the hypotenuse is equal to 1, $\cos \theta =$ adjacent side of the right-angled triangle, i.e., x -coordinate of point P. Similarly, y -coordinate of point P will be $\text{Sin } \theta$.



As θ approaches 90° , y -coordinate, i.e, $\sin \theta$ becomes 1 and x -coordinate, i.e., $\cos \theta$ becomes 0. The value of $\text{Cos } 90^\circ = 0$ and $\text{Sin } 90^\circ = 1$. Therefore, the value of

$$\text{Tan } 90^\circ = \frac{\sin \theta}{\cos \theta} = \frac{1}{0} = \text{Not defined} .$$

Value of Tan 90° using trigonometric identities

$\tan 90^\circ$ can be represented using trigonometric identities as follows:

- $\frac{\sin(90^\circ)}{\cos(90^\circ)}$
- $\frac{\sin 90^\circ}{\sqrt{1-\sin^2 90^\circ}}$
- $\frac{\sqrt{1-\cos^2 90^\circ}}{\cos 90^\circ}$
- $\frac{1}{\sqrt{\operatorname{Cosec}^2 90^\circ - 1}}$
- $\sqrt{\operatorname{Sec}^2 90^\circ - 1}$
- $\frac{1}{\cot 90^\circ}$

Examples to Make It Easy

Example 1: Find the value of $\tan 90^\circ + \cot 90^\circ$

Solution: $\tan 90^\circ + \cot 90^\circ = \text{Undefined} + 0 = \text{Undefined}$

Example 2: Solve $\tan^2 90^\circ - \sec^2 90^\circ$.

Solution: Since $\tan 90^\circ$ and $\sec 90^\circ$ are undefined, the expression remains undefined.

Example 3: A vertical flagpole casts no shadow at noon. What is the angle of elevation of the sun, and how does it relate to $\tan 90^\circ$?

Solution: At noon, the sun is directly overhead, making the angle of elevation 90° . The tangent function at this angle is undefined because the shadow (adjacent side) is zero.

Practice Questions

Test yourself with these problems:

1. Using the identity $1 + \tan^2 \theta = \sec^2 \theta$, explain why $\tan 90^\circ$ is undefined.
2. Find $\tan 90^\circ \times \cot 90^\circ$.
3. A rocket launches vertically. What is the angle between the rocket's path and the ground?

Real-Life Applications of $\tan 90^\circ$:

1. **Rocket Launches:** When a rocket takes off vertically, its path makes a 90° angle with the ground.
2. **Architecture:** The concept of verticality in buildings relies on perpendicular angles (90°) for stability.
3. **Sun Elevation:** At solar noon, the sun is directly overhead, forming a 90° angle with the ground.
4. **Surveying:** Theodolites measure vertical angles, including 90° , for height and distance calculations.

5. **Physics of Free Fall:** When an object falls straight down, it moves at an angle of 90° relative to the ground.

Conclusion

Tan 90° is a unique trigonometric function that remains undefined because it involves division by zero. While it doesn't have a finite value, its concept is fundamental in physics, engineering, and everyday scenarios. Understanding tan 90° helps solve practical problems in motion analysis, surveying, and architecture.

Frequently asked questions (FAQs)

Q1. How much does tan 90° mean?

Since it involves division by zero, tan 90° is undefinable.

Q2. What makes tan 90° ambiguous?

Since the adjacent side of a right triangle becomes zero at 90 degrees, the triangle is undefined.

Q3. What is the practical application of tan 90° ?

It is employed in the construction of constructions, rocket launches, and vertical height calculations.

Q4. Does the radian value of tan 90° change?

No, tan 90° is still undefined and is the same as $\tan\left(\frac{\pi}{2}\right)$.

Q5. What is tan 90° 's reciprocal?

Tan 90° has a reciprocal of cot 90° , or 0.