Solutions
Grade 10 Maths
Chapter 8: Quadratic Equations

Exercise 8.1

Q1. Which of the following are quadratic equations?

(i) $x^2+6x-4=0$

(ii) $\sqrt{3}x^2-2x+\frac{1}{2}=0$

(iii) $x^2+\frac{1}{x^2}=5$

(iv) $x-\frac{3}{x}=x^2$

(v) $2x^2-\sqrt{3x}+9=0$

(vi) $x^2-2x-\sqrt{x}-5=0$

(vii) $3x^2-5x+9=x^2-7x+3$

(viii) $x+\frac{1}{x}=1$

(ix) $x^2-3x=0$

(x) $\left(x+\frac{1}{x}\right)^2=3\left(x+\frac{1}{x}\right)+4$

(xi) $(2x+1)(3x+2)=6(x-1)(x-2)$

(xii) $x+\frac{1}{x}=x^2, x\neq 0$

(xiii) $16x^2-3=(2x+5)(5x-3)$

(xiv) $(x+2)^3=x^3-4$

(xv) $x(x+1)+8=(x+2)(x-2)$

Solution:

A polynomial equation is a quadratic equation, if it is of the form $ax^2+bx+c=0$ such that $a\neq 0$

(i) $x^2+6x-4=0$

It is a quadratic equation.

(ii) $\sqrt{3}x^2-2x+\frac{1}{2}=0$

It is a quadratic equation.

(iii) $x^2+\frac{1}{x^2}=5$

$\Rightarrow x^4-5x^2+1=0$

It is not a quadratic equation as the highest power of x is ' 4 '.

(iv) $x-\frac{3}{x}=x^2$

$\Rightarrow x^2-3=x^3$

It is not a quadratic equation.

(v) $2x^2-\sqrt{3x}+9=0$

It is not a quadratic equation as $\sqrt{x}$ is present instead of ' x '.

(vi) $x^2-2x-\sqrt{x}-5=0$

It is not a quadratic equation as an additional $\sqrt{x}$ term is present.

(vii) $3x^2-5x+9=x^2-7x+3$

$\Rightarrow 2x^2+2x+6=0$

It is a quadratic equation.

(viii) $x+\frac{1}{x}=1$

$\Rightarrow x^2+1-x=0$

It is a quadratic equation.

(ix) $x^2-3x=0$

It is a quadratic equation.

(x) $\left(x+\frac{1}{x}\right)^2=3\left(x+\frac{1}{x}\right)+4$

$\Rightarrow x^2+\frac{1}{x^2}+2=3x+\frac{3}{x}+4$

$\Rightarrow x^4+1+2x^2=3x^3+3x+4x^2$

It is not a quadratic equation.

(xi) $(2x+1)(3x+2)=6(x-1)(x-2)$

$\Rightarrow 6x^2+7x+2=6x^2-18x+12$

$\Rightarrow 25x=10$

It is not a quadratic equation.

(xii) $x+\frac{1}{x}=x^2, x\neq 0$

$\Rightarrow x^2+1=x^3$

It is not a quadratic equation.

(xiii) $16x^2-3=(2x+5)(5x-3)$

$\Rightarrow 16x^2-3=10x^2+19x-15$

$\Rightarrow 6x^2-19x+12=0$

It is a quadratic equation.

(xiv) $(x+2)^3 = x^3 - 4$
$\Rightarrow x^3 + 8 + 6x^2 + 12x = x^3 - 4$
$\Rightarrow 6x^2 + 12x + 12 = 0$
It is a quadratic equation.

(xv)$x(x+1) + 8 = (x+2)(x-2)$
$\Rightarrow x^2 + x + 8 = x^2 - 4$
$\Rightarrow x = -12$
It is not a quadratic equation.

Q2. In each of the following, determine whether the given values are solutions of the given equation or not:
(i) $x^2 - 3x + 2 = 0, x = 2, x = 1$
(ii) $x^2 + x + 1 = 0, x = 0, x = 1$
(iii) $x^2 - 3\sqrt{3}x + 6 = 0, x = \sqrt{3}, x = -2\sqrt{3}$
(iv) $x + \frac{1}{x} = \frac{13}{6}, x = \frac{5}{6}, x = \frac{4}{3}$
(v) $2x^2 - x + 9 = x^2 + 4x + 3, x = 2, x = 3$
(vi) $x^2 - \sqrt{2}x - 4 = 0, x = -\sqrt{2}, x = -2\sqrt{2}$
(vii) $a^2x^2 - 3abx + 2b^2 = 0, x = \frac{a}{b}, x = \frac{b}{a}$

Solution:

We will have to check for each value and see whether it satisfies the equation.
(i) $x^2 - 3x + 2 = 0, x = 2, x = 1$
For $x = 2$,
$= 2^2 - 3 \times 2 + 2$
$= 0$
Thus, $x = 2$ is a solution.
For, $x = 1$
$= 1^2 - 3 \times 1 + 2$
$= 0$
Thus, $x = 1$ is a solution.

(ii) $x^2 + x + 1 = 0, x = 0, x = 1$
For $x = 0$,
$= 0 + 0 + 1$
$= 1 \neq 0$ thus $x = 0$ is not a solution
For $x = 1$,
$= 1 + 1 + 1$
$= 3 \neq 0$ thus $x = 1$ is not a solution.

(iii) $x^2 - 3\sqrt{3}x + 6 = 0, x = \sqrt{3}, x = -2\sqrt{3}$

For $x = \sqrt{3}$

$= 3 - 3\sqrt{3} \times \sqrt{3} + 6$

$= 3 - 9 + 6$

$= 0$

Thus, $x = \sqrt{3}$ is a solution

For $x = -2\sqrt{3}$

$= (-2\sqrt{3})^2 - 3\sqrt{3} \times -2\sqrt{3} + 6$

$= 4 \times 3 + 18 + 6$

$= 36 \neq 0$

thus $x = -2\sqrt{3}$ is not a solution.

(iv) $x + \frac{1}{x} = \frac{13}{6}, x = \frac{5}{6}, x = \frac{4}{3}$

For $x = \frac{5}{6}$

$\Rightarrow \frac{5}{6} + \frac{6}{5} = \frac{13}{6}$

$\Rightarrow \frac{61}{30} \neq \frac{13}{6}$

thus $x = \frac{5}{6}$ is not a solution

For $x = \frac{4}{3}$

$\Rightarrow \frac{4}{3} + \frac{3}{4} = \frac{13}{6}$

$\Rightarrow \frac{25}{12} \neq \frac{13}{6}$, thus $x = \frac{4}{3}$ is not a solution.

(v) $2x^2 - x + 9 = x^2 + 4x + 3, x = 2, x = 3$

For $x = 2$,

$\Rightarrow 2 \times 4 - 2 + 9 = 15$

$\Rightarrow 4 + 4 \times 2 + 3 = 15$

$\Rightarrow 15 = 15$, thus $x = 2$ is a solution.

For $x = 3$

$\Rightarrow 2 \times 9 - 3 + 9 = 9 + 4 \times 3 + 3$

$\Rightarrow 24 = 24$, thus $x = 3$ is also a solution.

(vi) $x^2 - \sqrt{2}x - 4 = 0, x = -\sqrt{2}, x = -2\sqrt{2}$

For $x = -\sqrt{2}$,

$\Rightarrow 2 - \sqrt{2} \times -\sqrt{2} - 4$

$\Rightarrow 2 + 2 - 4$

$\Rightarrow 0$

Thus, $x = -\sqrt{2}$ is a solution

For $x = -2\sqrt{2}$

$\Rightarrow 4 \times 2 - \sqrt{2} \times -2\sqrt{2} - 4$

$\Rightarrow 8 + 8 - 4$

$\Rightarrow 12 \neq 0$, thus $x = -2\sqrt{2}$ is not a solution.

(vii) $a^2x^2 - 3abx + 2b^2 = 0, x = \frac{a}{b}, x = \frac{b}{a}$

For, $x = \frac{a}{b}$

$\Rightarrow a^2 \times \frac{a^2}{b^2} - 3ab \times \frac{a}{b} + 2 \times b^2$

$\Rightarrow \frac{a^4}{b^2} - 3a^2 + 2b^2 \neq 0$, thus $x = \frac{a}{b}$ is not a solution

For $x = \frac{\mathrm{b}}{\mathrm{a}}$

$\Rightarrow a^2 \times \frac{b^2}{a^2} - 3ab \times \frac{b}{a} + 2b^2 = 0$

$\Rightarrow b^2 - 3b^2 + 2b^2 = 0$

$\Rightarrow 0 = 0$, thus $x = \frac{b}{a}$ is a solution.

Q3. In each of the following, find the value of k for which the given value is a solution of the given equation:

(i) $7x^2 + kx - 3 = 0, x = \frac{2}{3}$

(ii) $x^2 - x(a + b) + k = 0, x = a$

(iii) $kx^2 + \sqrt{2}x - 4 = 0, x = \sqrt{2}$

(iv) $x^2 + 3ax + k = 0, x = -a$

Solution:

For the given value to be a solution, it should satisfy the quadratic equation

(i) $7x^2 + kx - 3 = 0, x = 2/3$

$\Rightarrow 7 \times \frac{4}{9} + k \times \frac{2}{3} - 3 = 0$

$\Rightarrow \frac{2k}{3} = 3 - \frac{28}{9} = -\frac{1}{9}$

$\Rightarrow k = -\frac{1}{6}$

(ii) $x^2 - x(a + b) + k = 0, x = a$

$\Rightarrow a^2 - a(a + b) + k = 0$

$\Rightarrow a^2 - a^2 - ab + k = 0$

$\Rightarrow k = ab$

(iii) $kx^2 + \sqrt{2}x - 4 = 0, x = \sqrt{2}$

$\Rightarrow k \times 2 + \sqrt{2} \times \sqrt{2} - 4 = 0$

$\Rightarrow 2k = 2$
$\Rightarrow k = 1$

(iv) $x^2 + 3ax + k = 0, x = -a$
$\Rightarrow a^2 - 3a \times a + k = 0$
$\Rightarrow k = 2a^2$

Q4. If $x = \frac{2}{3}$ and $x = -3$ are the roots of the equation $ax^2 + 7x + b = 0$, find the values of a and b.

Solution:

Given: If $x = \frac{2}{3}$ and $x = -3$ are the roots of the equation $ax^2 + 7x + b = 0$
To find: the values of a and b.
Solution: Quadratic equation in roots form:
$(x - a)(x - b) = 0$, where a and b are the roots
Given, $x = \frac{2}{3}$ and $x = -3$ are the roots of the equation $ax^2 + 7x + b = 0$
Quadratic equation is,

$$\left(x - \frac{2}{3}\right)(x + 3) = 0$$

$$\Rightarrow x^2 - \frac{2x}{3} + 3x - 2 = 0$$

$$\Rightarrow \frac{3x^2 - 2x + 9x - 6}{3} = 0$$

$$\Rightarrow 3x^2 + 7x - 6 = 0$$

On comparing with $ax^2 + 7x + b = 0$
We get, $a = 3$ and $b = -6$.

Q5. Determine, if 3 is a root of the equation given below:

$$\sqrt{x^2 - 4x + 3} + \sqrt{x^2 - 9} = \sqrt{4x^2 - 14x + 16}$$

Solution:

For the given value to be a root, it should satisfy given equation

$$\sqrt{x^2 - 4x + 3} + \sqrt{x^2 - 9} = \sqrt{4x^2 - 14x + 16}$$

$$\Rightarrow \sqrt{3^2 - 4 \times 3 + 3} + \sqrt{3^2 - 9} = \sqrt{4 \times 3^2 - 14 \times 3 + 16}$$

$$\Rightarrow 0 + 0 = \sqrt{10}$$

Thus $x = 3$ does not satisfy the given equation and it is not a root of the equation.

Exercise 8.2

Q1. The product of two consecutive positive integers is 306. Form the quadratic equation to find the integers, if x denoted the smaller integer.

Solution:

Let the consecutive numbers be ' a ' and ' $a+1$ ' respectively.

Given, product of two consecutive positive integers is 306

$a \times (a+1) = 306$

$\Rightarrow a^2 + a - 306 = 0.$

Q2. John and Jivanti together have 45 marbles. Both of them lost 5 marbles each, and the product of the number of marbles they now have is 128. Form the quadratic equation to find how many marbles they had to start with, if john had x marbles.

Solution:

Number of marbles John has is x .

Given, John and Jivanti together have 45 marbles.

Number of marbles which Jivanti has $= 45 - x$

Now, both of them lost 5 marbles each, and the product of the number of marbles they now have is 128.

So John will have $x - 5$ marbles and Jivanti will have $45 - x - 5 = 40 - x$ marbles.

$\Rightarrow (x-5)(40-x) = 128$

$\Rightarrow 40x - 200 + 5x - x^2 = 128$

$\Rightarrow 40x - 200 + 5x - x^2 - 128 = 0$

$\Rightarrow 45x - 328 - x^2 = 0$

$\Rightarrow x^2 - 45x + 328 = 0.$

Q3. A cottage industry produces a certain number of toys in a day. The cost of production of each toy (in rupees) was found to be 55 minus the number of articles produced in a day. On a particular day, the total cost of production was ₹750. If x denotes the number of toys produced that day, form the quadratic equation to find x.

Solution:

Number of toys produced that day is ' x '.

Cost of production of each toy (in rupees) was found to be 55 minus the number of articles produced in a day.

$\therefore$ Cost of production of each toy $= 55 - x$

Given, total cost of production $=$ ₹750

$\Rightarrow x \times (55 - x) = 750$

$\Rightarrow -x^2 + 55x - 750 = 0$

$\Rightarrow x^2 - 55x + 750 = 0.$

Q4. The height of the right TRIANGLE IS 7 CM LESS THAN ITS BASE. If the hypotenuse is 13 cm, form the quadratic equation to find the base of the triangle.

Solution:

By Pythagoras theorem :

$Hypotenuse^2 = perpendicular^2 + base^2$

Given, height of a right TRIANGLE IS 7 CM LESS THAN ITS BASE and the hypotenuse is 13 cm .

Let the base be ' x '

$\Rightarrow 13^2 = (x-7)^2 + x^2$

$\Rightarrow 169 = x^2 - 14x + 49 + x^2$

$\Rightarrow x^2 - 7x - 60 = 0.$

Q5. An express train takes 1 hour less than a passenger train to travel 132 km between Mysore and Bangalore. If the average speed of the express train is 11 km/hr more than that of the passenger train, form the quadratic equation to find the average speed of express train.

Solution:

Given: An express train takes 1 hour less than a passenger train to travel 132 km between Mysore and Bangalore. If the average speed of the express train is 11 km/hr more than that of the passenger train.

To find: the quadratic equation to find the average speed of express train.

Solution: Let the average speed of passenger train be ' x ' km/hr and the time taken by passenger train be ' t' hr .

Since an express train takes 1 hour less than a passenger train and the average speed of the express train is 11 km/hr more than that of the passenger train.

So, For the express train average speed $= x + 11$, time taken $= t - 1$

Since, Distance = speed × time

Given, total distance traveled = 132 km

For passenger train:

$\Rightarrow x \times \mathrm{t} = 132$

$\Rightarrow t = \dfrac{132}{x}$

Also for express train $(x+11) \times (t-1) = 132$

Substitute the value of t from (1),

$(x+11)\left(\dfrac{132}{x} - 1\right) = 132$

$\Rightarrow (x+11)(132-x) = 132x$

$\Rightarrow 132x - x^2 + 1452 - 11x = 132x$

$\Rightarrow -x^2 - 11x + 1452 = 0$

$\Rightarrow x^2 + 11x - 1452 = 0$ is the required quadratic equation.

Q6. A train travels 360 km at a uniform speed. If the speed had been 5 km/hr more, it would have taken 4 hour less for the same journey. Form the quadratic equation to find the speed of the train.

Solution:

Let the speed of the train be $'a'$ km/hr and the actual time taken be $'t'$

Given, train travels 360 km at a uniform speed. If the speed had been 5 km/hr more, it would have taken 4 hour less for the same journey.

Distance = speed × time

$\Rightarrow 360 = a \times t$

$\Rightarrow t = \dfrac{360}{a}$

Also, $360 = (a + 5)(t - 4)$

$\Rightarrow 360 = (a + 5)\left(\dfrac{360}{a} - 4\right)$

$\Rightarrow 360a = (a + 5)(360 - 4a)$

$\Rightarrow 360a = 360a + 1800 - 4a^2 - 20a$

$\Rightarrow a^2 + 5a - 450 = 0$

Upon solving we will get a to be 18.86 and −23.86 but the speed can't be negative so the speed of the train is 18.86 km/hr.

Exercise 8.3

Q1. Solve the following quadratic equations by factorization:

$(x - 4)(x + 2) = 0$

Solution:

$(x - 4)(x + 2) = 0$ is already factorized

$\Rightarrow x = 4, -2.$

Q2. Solve the following quadratic equations by factorization:

$(2x + 3)(3x - 7) = 0$

Solution:

$(2x + 3)(3x - 7) = 0$ is already factorized

$\Rightarrow x = -\frac{3}{2}, \frac{7}{3}.$

Q3. Solve the following quadratic equations by factorization:

$x^2 - (\sqrt{3} + 1)x + \sqrt{3} = 0$

Solution:

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$x^2 - (\sqrt{3} + 1)x + \sqrt{3} = 0$

$\Rightarrow x^2 - x - \sqrt{3}x + \sqrt{3} = 0$
$\Rightarrow x(x-1) - \sqrt{3}(x-1) = 0$
$\Rightarrow (x-\sqrt{3})(x-1) = 0$
$\Rightarrow x = \sqrt{3}, 1.$

Q4. Solve the following quadratic equations by factorization:
$9x^2 - 3x - 2 = 0$
Solution:
In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.
$9x^2 - 3x - 2 = 0$
$\Rightarrow 9x^2 - 6x + 3x - 2 = 0$
$\Rightarrow 3x(3x-2) + (3x-2) = 0$
$\Rightarrow (3x+1)(3x-2) = 0$
$\Rightarrow x = -\frac{1}{3}, \frac{2}{3}.$

Q5. Solve the following quadratic equations by factorization:
$3\sqrt{5}x^2 + 25x - 10\sqrt{5} = 0$
Solution:
In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.
$3\sqrt{5}x^2 + 25x - 10\sqrt{5} = 0$
$\Rightarrow 3\sqrt{5}x^2 + 30x - 5x - 10\sqrt{5} = 0$
$\Rightarrow 3\sqrt{5}x(x + 2\sqrt{5}) - 5(x + 2\sqrt{5}) = 0$
$\Rightarrow (x + 2\sqrt{5})(3\sqrt{5}x - 5) = 0$
$\Rightarrow x = -2\sqrt{5}, \frac{\sqrt{5}}{3}.$

Q6. Solve the following quadratic equations by factorization:
$6x^2 + 11x + 3 = 0$
Solution:
In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.
$6x^2 + 11x + 3 = 0$
$\Rightarrow 6x^2 + 9x + 2x + 3 = 0$
$\Rightarrow 3x(2x+3) + (2x+3) = 0$
$\Rightarrow (3x+1)(2x+3) = 0$

$\Rightarrow x = -\frac{1}{3}, -\frac{3}{2}$.

Q7. Solve the following quadratic equations by factorization:
$5x^2 - 3x - 2 = 0$

Solution:

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$5x^2 - 3x - 2 = 0$
$\Rightarrow 5x^2 - 5x + 2x - 2 = 0$
$\Rightarrow 5x(x-1) + 2(x-1) = 0$
$\Rightarrow (5x+2)(x-1) = 0$
$\Rightarrow x = -\frac{2}{5}, 1$.

Q8. Solve the following quadratic equations by factorization:
$48x^2 - 13x - 1 = 0$

Solution:

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$48x^2 - 13x - 1 = 0$
$\Rightarrow 48x^2 - 16x + 3x - 1 = 0$
$\Rightarrow 16x(3x-1) + (3x-1) = 0$
$\Rightarrow (16x+1)(3x-1) = 0$
$\Rightarrow x = -\frac{1}{16}, \frac{1}{3}$.

Q9. Solve the following quadratic equations by factorization:
$3x^2 = -11x - 10$

Solution:

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$3x^2 = -11x - 10$
$\Rightarrow 3x^2 + 11x + 10 = 0$
$\Rightarrow 3x^2 + 6x + 5x + 10 = 0$
$\Rightarrow 3x(x+2) + 5(x+2) = 0$
$\Rightarrow (3x+5)(x+2) = 0$
$\Rightarrow x = -\frac{5}{3}, -2$.

Q10. Solve the following quadratic equations by factorization:

$25x(x+1)=-4$

Solution:

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$25x(x+1)=-4$

$\Rightarrow 25x^2+25x+4=0$

$\Rightarrow 25x^2+20x+5x+4=0$

$\Rightarrow 5x(5x+4)+(5x+4)=0$

$\Rightarrow (5x+1)(5x+4)=0$

$\Rightarrow x=-\frac{1}{5},-\frac{4}{5}.$

Q11. Solve the following quadratic equations by factorization:

$16x-\frac{10}{x}=27$

Solution:

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$16x-\frac{10}{x}=27$

$\Rightarrow 16x^2-27x-10=0$

$\Rightarrow 16x^2-32x+5x-10=0$

$\Rightarrow 16x(x-2)+5(x-2)=0$

$\Rightarrow (16x+5)(x-2)=0$

$\Rightarrow x=-\frac{5}{16},2.$

Q12. Solve the following quadratic equations by factorization:

$\sqrt{3}x^2-2\sqrt{2}x-2\sqrt{3}=0$

Solution:

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$\sqrt{3}x^2-2\sqrt{2}x-2\sqrt{3}=0$

$\Rightarrow \sqrt{3}x^2-3\sqrt{2}x+\sqrt{2}x-2\sqrt{3}=0$

$\Rightarrow \sqrt{3}x^2-\sqrt{3}\times\sqrt{3}x\sqrt{2}x+\sqrt{2}x-2x\sqrt{3}\times\sqrt{3}=0$

$\Rightarrow \sqrt{3}x^2-\sqrt{3}\times\sqrt{6}x+\sqrt{2}x-\sqrt{6}x\sqrt{3}=0$

$\Rightarrow \sqrt{3}x(x-\sqrt{6})+\sqrt{2}(x-\sqrt{6})=0$

$\Rightarrow (\sqrt{3}x+\sqrt{2})(x-\sqrt{6})=0$

$\Rightarrow x=\sqrt{6},-\sqrt{\left(\frac{2}{3}\right)}.$

Q13. Solve the following quadratic equations by factorization:

$4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$

Solution:

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$

$\Rightarrow 4\sqrt{3}x^2 + 8x - 3x - 2\sqrt{3} = 0$

$\Rightarrow 4x(\sqrt{3}x + 2) - \sqrt{3}(\sqrt{3}x + 2) = 0$

$\Rightarrow (4x - \sqrt{3})(\sqrt{3}x + 2) = 0$

$\Rightarrow x = \frac{\sqrt{3}}{4}, -\frac{2}{\sqrt{3}}.$

Q14. Solve the following quadratic equations by factorization:

$\sqrt{2}x^2 - 3x - 2\sqrt{2} = 0$

Solution:

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$\sqrt{2}x^2 - 3x - 2\sqrt{2} = 0$

$\Rightarrow \sqrt{2}x^2 - 4x + x - 2\sqrt{2} = 0$

$\Rightarrow \sqrt{2}x(x - 2\sqrt{2}) + (x - 2\sqrt{2}) = 0$

$\Rightarrow (\sqrt{2}x + 1)(x - 2\sqrt{2}) = 0$

$\Rightarrow x = -\frac{1}{\sqrt{2}}, 2\sqrt{2}.$

Q15. Solve the following quadratic equations by factorization:

$a^2x^2 - 3abx + 2b^2 = 0$

Solution:

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$a^2x^2 - 3abx + 2b^2 = 0$

$\Rightarrow a^2x^2 - 2abx - abx + 2b^2 = 0$

$\Rightarrow ax(ax - 2b) - b(ax - 2b) = 0$

$\Rightarrow (ax - b)(ax - 2b) = 0$

$\Rightarrow x = \frac{b}{a}, \frac{2b}{a}.$

Q16. Solve the following quadratic equations by factorization:

$x^2 - (\sqrt{2} + 1)x + \sqrt{2} = 0$

Solution:

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$x^2 - (\sqrt{2} + 1)x + \sqrt{2} = 0$

$\Rightarrow x^2 - \sqrt{2}x - x + \sqrt{2} = 0$

$\Rightarrow x(x - \sqrt{2}) - (x - \sqrt{2}) = 0$

$\Rightarrow (x - 1)(x - \sqrt{2}) = 0$

$\Rightarrow x = 1, \sqrt{2}.$

Q17. Solve the following quadratic equations by factorization:

$9x^2 - 6b^2x - (a^4 - b^4) = 0$

Solution:

Given: $9x^2 - 6b^2x - (a^4 - b^4) = 0$

To find: The value of above equation.

Solution: In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

We know: $a^2 - b^2 = (a + b)(a - b)$

Consider,

$9x^2 - 6b^2x - (a^4 - b^4) = 0$

Here,

$a^4 - b^4 = (a^2)^2 - (b^2)^2$

apply the above formula and solve,

$\Rightarrow 9x^2 - 3(a^2 + b^2)x + 3(a^2 - b^2)x - (a^2 + b^2)(a^2 - b^2) = 0$

$\Rightarrow 3x\left(3x - (a^2 + b^2)\right) + (a^2 - b^2)\left(3x - (a^2 + b^2)\right) = 0$

$\Rightarrow (3x + a^2 - b^2)(3x - (a^2 + b^2) = 0$

$\Rightarrow x = \frac{b^2 - a^2}{3}, \frac{a^2 + b^2}{3}.$

Q18. Solve the following quadratic equations by factorization:

$4x^2 + 4bx - (a^2 - b^2) = 0$

Solution:

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$\Rightarrow 4x^2 + 2(a + b)x - 2(a - b)x - (a - b)(a + b) = 0$

$\Rightarrow 2x(2x + a + b) - (a - b)(2x - (a + b)) = 0$

$\Rightarrow (2x - (a - b))(2x + a + b) = 0$

$$\Rightarrow x = \frac{a - b}{2}, -\frac{a + b}{2}$$

Q19. Solve the following quadratic equations by factorization:

$ax^2 + (4a^2 - 3b)x - 12ab = 0$

Solution:

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$ax^2 + (4a^2 - 3b)x - 12ab = 0$

$\Rightarrow ax^2 + 4a^2x - 3bx - 12ab = 0$

$\Rightarrow ax(x + 4a) - 3b(x + 4a) = 0$

$\Rightarrow (ax - 3b)(x + 4a) = 0$

$\Rightarrow x = \frac{3b}{a}, -4a$

Q20. Solve the following quadratic equations by factorization:

$2x^2 + ax - a^2 = 0$

Solution:

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$2x^2 + ax - a^2 = 0$

$\Rightarrow 2x^2 + 2ax - ax - a^2 = 0$

$\Rightarrow 2x(x + a) - a(x + a) = 0$

$\Rightarrow (2x - a)(x + a) = 0$

$\Rightarrow x = \frac{a}{2}, -a$

Q21. Solve the following quadratic equations by factorization:

$x^2 - 4\sqrt{2x} + 6 = 0$

Solution:

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$x^2 - 4\sqrt{2x} + 6 = 0$

$\Rightarrow x^2 - 3\sqrt{2}x - \sqrt{2}x + 6 = 0$

$\Rightarrow x(x - 3\sqrt{2}) - \sqrt{2}(x - 3\sqrt{2}) = 0$

$\Rightarrow (x - \sqrt{2})(x - 3\sqrt{2}) = 0$

$\Rightarrow x = \sqrt{2}, 3\sqrt{2}$

Q22. Solve the following quadratic equations by factorization:

$\frac{x+3}{x+2} = \frac{3x-7}{2x-3}$

Solution:

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$\frac{x+3}{x+2}=\frac{3x-7}{2x-3}$

$\Rightarrow 2x^2-9+3x=3x^2-14-x$

$\Rightarrow x^2-4x-5=0$

$\Rightarrow x^2-5\mathrm{x}+\mathrm{x}-5=0$

$\Rightarrow (x-5)(x+1)=0$

$\Rightarrow x=-1,5$

Q23. Solve the following quadratic equations by factorization:

$\frac{2x}{x-4}+\frac{2x-5}{x-3}=\frac{25}{3}$

Solution:

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$\frac{2x}{x-4}+\frac{2x-5}{x-3}=\frac{25}{3}$

$\Rightarrow 3\times(2x^2-6x+2x^2+20-13x)=25(x-4)(x-3)$

$\Rightarrow 12x^2-57x+60=25x^2+300-175x$

$\Rightarrow 13x^2-118x+240=0$

$\Rightarrow 13x^2-78x-40x+240=0$

$\Rightarrow 13x(x-6)-40(x-6)=0$

$\Rightarrow (13x-40)(x-6)=0$

$\Rightarrow x=6,\frac{40}{13}.$

Q24. Solve the following quadratic equations by factorization:

$\frac{x+3}{x-2}-\frac{1-x}{x}=\frac{17}{4}$

Solution:

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$\frac{x+3}{x-2}-\frac{1-x}{x}=\frac{17}{4}$

$\Rightarrow 4(x^2+3x+(x-1)(x-2))=17(x^2-2x)$

$\Rightarrow 4(x^2+3x+x^2-3x+2)=17(x^2-2x)$

$\Rightarrow 8x^2+8=17x^2-34x$

$\Rightarrow 9x^2-34x-8=0$

$\Rightarrow 9x^2 - 36x + 2x - 8 = 0$
$\Rightarrow 9x(x-4) + 2(x-4) = 0$
$\Rightarrow (9x+2)(x-4) = 0$
$\Rightarrow x = -\frac{2}{9}, 4$

Q25. Solve the following quadratic equations by factorization:
$\frac{x-3}{x+3} - \frac{x+3}{x-3} = \frac{48}{7}, x \neq 3, x \neq -3$

Solution:

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$\frac{x-3}{x+3} - \frac{x+3}{x-3} = \frac{48}{7}, x \neq 3, x \neq -3$

Taking L.C.M

$\frac{(x-3)(x-3)+(x+3)(x+3)}{(x-3)(x+3)} = \frac{48}{7}$

$\frac{x^2+9-6x+x^2+9+6x}{x^2-9} = \frac{48}{7}$

$\frac{2x^2+18}{x^2-9} = \frac{48}{7}$

Cross Multiplying we get,

$\Rightarrow 7(2x^2+18) = 48(x^2-9)$
$\Rightarrow -84x = 48x^2 - 432$
$\Rightarrow 4x^2 + 7x - 36 = 0$
$\Rightarrow 4x^2 + 16x - 9x - 36 = 0$
$\Rightarrow 4x(x+4) - 9(x+4) = 0$
$\Rightarrow (4x-9)(x+4) = 0$
$\Rightarrow x = \frac{9}{4}, -4$

Q26. Solve the following quadratic equations by factorization:
$\frac{1}{x-2} + \frac{2}{x-1} = \frac{6}{x}, x \neq 0$

Solution:

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$\frac{1}{x-2} + \frac{2}{x-1} = \frac{6}{x}, x \neq 0$

$\Rightarrow x(x-1+2x-4) = 6(x^2-3x+2)$
$\Rightarrow 3x^2 - 5x = 6x^2 - 18x + 12$

$\Rightarrow 3x^2 - 13x + 12 = 0$
$\Rightarrow 3x^2 - 9x - 4x + 12 = 0$
$\Rightarrow 3x(x-3) - 4(x-3) = 0$
$\Rightarrow (3x-4)(x-3) = 0$
$\Rightarrow x = \frac{4}{3}, 3.$

Q27. Solve the following quadratic equations by factorization:
$\frac{x+1}{x-1} - \frac{x-1}{x+1} = \frac{5}{6}, x \neq 1, -1$

Solution:

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$\frac{x+1}{x-1} - \frac{x-1}{x+1} = \frac{5}{6}, x \neq 1, -1$
$\Rightarrow 6((x+1)^2 - (x-1)^2) = 5(x^2 - 1)$
$\Rightarrow 6 \times 4x = 5x^2 - 5$
$\Rightarrow 5x^2 - 24x - 5 = 0$
$\Rightarrow 5x^2 - 25x + x - 5 = 0$
$\Rightarrow 5x(x-5) + 1(x-5) = 0$
$\Rightarrow (5x+1)(x-5) = 0$
$\Rightarrow x = 5, -\frac{1}{5}.$

Q28. Solve the following quadratic equations by factorization:
$\frac{x-1}{2x+1} + \frac{2x+1}{x-1} = \frac{5}{2}, x \neq -\frac{1}{2}, 1$

Solution:

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$\frac{x-1}{2x+1} + \frac{2x+1}{x-1} = \frac{5}{2}, x \neq -\frac{1}{2}, 1$
$\Rightarrow 2(x^2 - 2x + 1 + 4x^2 + 4x + 1) = 5(2x^2 - x - 1)$
$\Rightarrow 10x^2 + 4x + 4 = 10x^2 - 5x - 5$
$\Rightarrow 9x = -9$
$\Rightarrow x = -1.$

Q29. Solve the following quadratic equations by factorization:
$3x^2 - 14x - 5 = 0$

Solution:

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$3x^2 - 14x - 5 = 0$

$\Rightarrow 3x^2 - 15x + x - 5 = 0$

$\Rightarrow 3x(x-5) + 1(x-5) = 0$

$\Rightarrow (3x+1)(x-5) = 0$

$\Rightarrow x = 5, -\frac{1}{3}$

Q30. Solve the following quadratic equations by factorization:

$\frac{m}{n}x^2 + \frac{n}{m} = 1 - 2x$

Solution:

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$\Rightarrow \frac{m}{n}x^2 + \frac{n}{m} = 1 - 2x$

$\Rightarrow m^2x^2 + n^2 = mn - 2mnx$

$\Rightarrow m^2x^2 + 2mnx - mn + n^2 = 0$

$\Rightarrow (mx+n)^2 = mn$

$\Rightarrow mx + n = \pm\sqrt{mn}$

$\Rightarrow x = \frac{-n\pm\sqrt{mn}}{m}$.

Q31. Solve the following quadratic equations by factorization:

$\frac{x-a}{x-b} + \frac{x-b}{x-a} = \frac{a}{b} + \frac{b}{a}$

Solution:

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$\frac{x-a}{x-b} + \frac{x-b}{x-a} = \frac{a}{b} + \frac{b}{a}$

Let $\frac{x-a}{x-b} = y$ and $\frac{a}{b} = c$

So $\frac{x-b}{x-a} = \frac{1}{y}$ and $\frac{b}{a} = \frac{1}{c}$

$\Rightarrow y + \frac{1}{y} = c + \frac{1}{c}$

$\Rightarrow \frac{y^2+1}{y} = \frac{c^2+1}{c}$

$\Rightarrow c(y^2+1) = y(c^2+1)$

$\Rightarrow cy^2 + c = c^2y + y$

$\Rightarrow cy^2 - c^2y - y + c = 0$
$\Rightarrow cy(y-c) - 1(y-c) = 0$
$\Rightarrow (cy-1)(y-c) = 0$
$\Rightarrow y = \frac{1}{c}, c$
From (1) $\Rightarrow \frac{x-a}{x-b} = c$ and $\frac{a}{b} = c$
$\Rightarrow (x-a)(x-b) = \frac{a}{b}$ and $(x-b)(x-a) = \frac{b}{a}$
$\Rightarrow x^2 - bx - ax + ab = \frac{a}{b}$
$\Rightarrow b(x^2 - bx - ax + ab) = a$
$\Rightarrow bx^2 - b^2x - abx + ab^2 = a$
$\Rightarrow bx(x-b) - ab(x-b) = a$
$\Rightarrow (bx - ab)(x-b) = a \Rightarrow (bx - ab) = a$ and $(x-b) = a$
$So, bx - ab = a \Rightarrow bx = a + ab \Rightarrow x = \frac{a+ab}{b}$ and $x - b = a \Rightarrow x = a + b$
$Hence\ x = \frac{a+ab}{b}, (a+b).$

Q32. Solve the following quadratic equations by factorization:
$$\frac{1}{(x-1)(x-2)} + \frac{1}{(x-2)(x-3)} + \frac{1}{(x-3)(x-4)} = \frac{1}{6}$$

Solution:
In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.
$$\frac{1}{(x-1)(x-2)} + \frac{1}{(x-2)(x-3)} + \frac{1}{(x-3)(x-4)} = \frac{1}{6}$$
$\Rightarrow (x-3)(x-4) + (x-1)(x-4) + (x-1)(x-2)$
$= \frac{(x-1)(x-2)(x-3)(x-4)}{6}$
$\Rightarrow (x^2 - 4x - 3x + 12) + (x^2 - 4x - x + 4) + (x^2 - 2x - x + 1)$
$= \frac{(x^2 - 2x - x + 2)(x^2 - 4x - 3x + 12)}{6}$
$\Rightarrow 6(x^2 + 12 - 7x + x^2 + 4 - 5x + x^2 - 3x + 2)$
$= (x^2 - 3x + 2)(x^2 + 12 - 7x)$
$\Rightarrow 6(3x^2 + 18 - 15x)$
$= x^4 + 12x^2 - 7x^3 - 3x^3 - 36x + 21x^2 + 2x^2 + 24 - 14x$
$\Rightarrow 18x^2 - 90x + 108$
$= x^4 + 12x^2 - 7x^3 - 3x^3 - 36x + 21x^2 + 2x^2 + 24 - 14x$
$\Rightarrow x^4 - 10x^3 + 17x^2 + 40x + 84 = 0$
Let $P(x) = x^4 - 10x^3 + 17x^2 + 40x + 84$
At $x = -2, (-2)^4 - 10(-2)^3 + 17(-2)^2 + 40(-2) + 84 = 16 + 80 + 68 - 80 +$

84
$P(x)=0$ therefore, $x+2$ is a factor of $P(x)$.On dividing $P(x)$ by $(x+2)$, we get $x^3-12x^2+41x-42$
Let $g(x)=x^3-12x^2+41x-42, P(x)=(x-2)g(x)$
at $x=-2 g(x)=0$ therefore, $x+2$ is a factor of $g(x)$. On dividing $g(x)$ by $(x+2)$, we get $x^2-14x+49$
Therefore, $P(x)=(x-2)(x-2)(x^2-14x+49)$ Using, $(a-b)^2=a^2+b^2-2ab$, we have $P(x)=(x+2)^2(x-7)^2$
$\Rightarrow (x+2)^2(x-7)^2=0$
Therefore, possible value of ' x ' are $-2,-2$, and $-7,-7$.

Q33. Solve the following quadratic equations by factorization:

$$(x-5)(x-6)=\frac{25}{(24)^2}$$

Solution:

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$$(x-5)(x-6)=\frac{25}{(24)^2}$$

$$\Rightarrow x^2-11x+30=\frac{25}{24}$$

$$\Rightarrow x^2-11x+\frac{121}{4}-\frac{121}{4}+30=\frac{25}{24}$$

$$\Rightarrow x^2-11x+\frac{121}{4}=\frac{25}{24}+\frac{1}{4}$$

$$\Rightarrow x^2-11x+\frac{121}{4}=\frac{676}{(4\times 24^2)}$$

$$\Rightarrow \left(x-\frac{11}{2}\right)^2=\left(\frac{13}{24}\right)^2$$

$$\Rightarrow x-\frac{11}{2}=\pm\frac{13}{24}$$

$$\Rightarrow x=\frac{11}{2}+\frac{13}{24} \text{ or } x=\frac{11}{2}-\frac{13}{24}$$

$$\Rightarrow x=\frac{145}{24}=6\frac{1}{24}, x=\frac{119}{24}=4\frac{23}{24}.$$

Q34. Solve the following quadratic equations by factorization:

$$7x+\frac{3}{x}=35\frac{3}{5}$$

Solution:

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$$7x+\frac{3}{x}=35\frac{3}{5}$$
$$\Rightarrow 7x^2+3=\frac{178x}{5}$$
$$\Rightarrow 35x^2-178x+15=0$$
$$\Rightarrow 35x^2-175x-3x+15=0$$
$$\Rightarrow 35x(x-5)-3(x-5)=0$$
$$\Rightarrow (35x-3)(x-5)=0$$
$$\Rightarrow x=5,\tfrac{3}{35}.$$

Q35. Solve the following quadratic equations by factorization:

$$\frac{a}{x-a}+\frac{b}{x-b}=\frac{2c}{x-c}$$

Solution:

Given: $\frac{a}{x-a}+\frac{b}{x-b}=\frac{2c}{x-c}$

To find: The value of x.

Solution: In factorization, we write the coefficient of middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the product of these two factors will be equal to the product of the coefficient of x^2 and the constant term.

$$\frac{a}{x-a}+\frac{b}{x-b}=\frac{2c}{x-c}$$

Take the LCM of the denominators.LCM is $(x-a)(x-b)(x-c)$

Now solve for (1),

$$\Rightarrow a(x-b)(x-c)+b(x-a)(x-c)=2c(x-a)(x-b)$$
$$\Rightarrow (a(x-b)+b(x-a))(x-c)=2c(x-a)(x-b)$$
$$\Rightarrow (ax-ab+bx-ab)(x-c)=2c(x^2-bx-ax+ab)$$
$$\Rightarrow (ax-ab+bx-ab)(x-c)=2cx^2-2cbx-2cax+2cab$$
$$\Rightarrow ((a+b)x-2ab)(x-c)=2cx^2-2c(a+b)x+2abc$$
$$\Rightarrow (a+b)x^2-(a+b)cx-2abx+2abc=2cx^2-2(a+b)cx+2abc$$
$$\Rightarrow (a+b-2c)x^2+((a+b)c-2ab)x=0$$
$$\Rightarrow x\big[(a+b-2c)x+\big((a+b)c-2ab\big)\big]=0$$
$$\Rightarrow x=0 and (a+b-2c)x+\big((a+b)c-2ab\big)=0$$
$$\Rightarrow (a+b-2c)x=-[(a+b)c-2ab)2ab]$$
$$\Rightarrow (a+b-2c)x=-(ac+bc-2ab)$$
$$\Rightarrow x=\frac{-(ac+bc-2ab)}{a+b-2c}$$
$$\Rightarrow x=\frac{2ab-ac-bc}{a+b-2c}$$

Hence $x=0,\frac{2ab-ac-bc}{a+b-2c}$.

Q36. Solve the following quadratic equations by factorization:

$x^2 + 2ab = (2a + b)x$

Solution:

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$x^2 + 2ab = (2a + b)x$

$\Rightarrow x^2 - (2a + b)x + 2ab = 0$

$\Rightarrow x^2 - 2ax - bx + 2ab = 0$

$\Rightarrow x(x - 2a) - b(x - 2a) = 0$

$\Rightarrow (x - b)(x - 2a) = 0$

$\Rightarrow x = b, 2a$

Q37. Solve the following quadratic equations by factorization:

$(a + b)^2x^2 - 4abx - (a - b)^2 = 0$

Solution:

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$(a + b)^2x^2 - 4abx - (a - b)^2 = 0$

$\Rightarrow (a + b)^2x^2 - (a^2 + b^2 + 2ab - a^2 - b^2 + 2ab)x - (a - b)^2 = 0$

$\Rightarrow (a + b)^2x^2 - (a + b)^2x + (a - b)^2x - (a - b)^2 = 0$

$\Rightarrow (a + b)^2x(x - 1) + (a - b)^2(x - 1) = 0$

$\Rightarrow ((a + b)^2x + (a - b)^2)(x - 1) = 0$

$\Rightarrow x = 1, -\frac{(a - b)^2}{(a + b)^2}$

Q38. Solve the following quadratic equations by factorization:

$a(x^2 + 1) - x(a^2 + 1) = 0$

Solution:

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$a(x^2 + 1) - x(a^2 + 1) = 0$

$\Rightarrow ax^2 + a - a^2x - x = 0$

$\Rightarrow ax^2 - x(a^2 + 1) + a = 0$

$\Rightarrow ax(x - a) - 1(x - a) = 0$

$\Rightarrow (ax - 1)(x - a) = 0$

$\Rightarrow x = \frac{1}{a}, a$

Q39. Solve the following quadratic equations by factorization:

$x^2 - x - a(a + 1) = 0$

Solution:

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$x^2 - x - a(a+1) = 0$

$\Rightarrow x^2 - a^2 - x - a = 0$

$\Rightarrow (x+a)(x-a) - 1(x+a) = 0$

$\Rightarrow (x+a)(x-a-1) = 0$

$\Rightarrow x = -a, a+1$

Q40. Solve the following quadratic equations by factorization:

$x^2 + \left(a + \frac{1}{a}\right)x + 1 = 0$

Solution:

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$x^2 + \left(a + \frac{1}{a}\right)x + 1 = 0$

$\Rightarrow ax^2 + a^2x + x + a = 0$

$\Rightarrow ax(x+a) + 1(x+a) = 0$

$\Rightarrow (ax+1)(x+a) = 0$

$\Rightarrow x = -a, -\frac{1}{a}$

Q41. Solve the following quadratic equations by factorization:

$abx^2 + (b^2 - ac)x - bc = 0$

Solution:

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$abx^2 + (b^2 - ac)x - bc = 0$

$\Rightarrow abx^2 - acx + b^2x - bc = 0$

$\Rightarrow ax(bx - c) + b(bx - c) = 0$

$\Rightarrow (ax+b)(bx-c) = 0$

$\Rightarrow x = -\frac{b}{a}, \frac{c}{b}.$

Q42. Solve the following quadratic equations by factorization:

$a^2b^2x^2 + b^2x - a^2x - 1 = 0$

Solution:

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$a^2b^2x^2 + b^2x - a^2x - 1 = 0$

$\Rightarrow b^2x(a^2x + 1) - (a^2x + 1) = 0$

$\Rightarrow (b^2x - 1)(a^2x + 1) = 0$

$\Rightarrow x = -\frac{1}{a^2}, \frac{1}{b^2}$

Q43. Solve the following quadratic equations by factorization:

Solve for x: $\frac{x-1}{x-2} + \frac{x-3}{x-4} = 3\frac{1}{3}, x \neq 2,4$

Solution:

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$$\frac{x-1}{x-2} + \frac{x-3}{x-4} = 3\frac{1}{3}, x \neq 2,4$$

$\Rightarrow 3(x-1)(x-4) + 3(x-2)(x-3) = 10(x-2)(x-4)$

$\Rightarrow 3x^2 + 12 - 15x + 3x^2 + 18 - 15x = 10x^2 - 60x + 80$

$\Rightarrow 4x^2 - 30x + 50 = 0$

$\Rightarrow 2x^2 - 15x + 25 = 0$

$\Rightarrow 2x^2 - 10x - 5x + 25 = 0$

$\Rightarrow 2x(x-5) - 5(x-5) = 0$

$\Rightarrow (2x-5)(x-5) = 0$

Thus, $x = \frac{5}{2}, 5$

Q44. Solve the following quadratic equations by factorization:

$3x^2 - 2\sqrt{6}x + 2 = 0$

Solution:

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$3x^2 - 2\sqrt{6}x + 2 = 0$

$\Rightarrow (\sqrt{3}x)^2 - 2\sqrt{6x} + (\sqrt{2})^2 = 0$

$\Rightarrow (\sqrt{3x} - \sqrt{2})^2 = 0$

$\Rightarrow x = \frac{\sqrt{2}}{\sqrt{3}}, \frac{\sqrt{2}}{\sqrt{3}}$

Q45. Solve the following quadratic equations by factorization:

$$\frac{1}{x-1} - \frac{1}{x+5} = \frac{6}{7}, x \neq 1, -5$$

Solution:

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$$\frac{1}{x-1}-\frac{1}{x+5}=\frac{6}{7}, x \neq 1,-5$$

$\Rightarrow 7(x+5-x+1)=6(x-1)(x+5)$

$\Rightarrow 42=6x^2+24x-30$

$\Rightarrow x^2+4x-12=0$

$\Rightarrow x^2+6x-2x-12=0$

$\Rightarrow x(x+6)-2(x+6)=0$

$\Rightarrow (x-2)(x+6)=0$

$\Rightarrow x=2,-6$

Q46. Solve the following quadratic equations by factorization:

$$\frac{1}{x}-\frac{1}{x-2}=3, x \neq 0,2$$

Solution:

Given: $\frac{1}{x}-\frac{1}{x-2}=3, x \neq 0,2$

To find: Solve the given quadratic equations by factorization.

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$$\frac{1}{x}-\frac{1}{x-2}=3, x \neq 0,2$$

$\Rightarrow x-2-x=3x(x-2)$

$\Rightarrow x-2-x=3(x^2-2x)$

$\Rightarrow x-2-x=3x^2-6x$

$\Rightarrow 3x^2-6x+2=0$

Now convert the terms in above equation involving 3, $\Rightarrow 3x^2-(3+3)x+(3-1)=0$

Now add and subtract $\sqrt{3}$ in the coefficient of x, $=3x^2-(3+\sqrt{3}+3-\sqrt{3})x+(3-1)=0$

Now $3=(\sqrt{3})^2$ and $1=1^2$

$\Rightarrow (\sqrt{3}x)^2-[(3+\sqrt{3})+(3-\sqrt{3})]x+(3-1)=0$

$\Rightarrow (\sqrt{3}x)^2-(3+\sqrt{3})x-(3-\sqrt{3})x+\left(\sqrt{3^2}-1^2\right)=0$

Apply the formula $a^2-b^2=(a+b)(a-b)$ in $\left(\sqrt{3^2}-1^2\right)$

$\Rightarrow (\sqrt{3}x)^2-(3+\sqrt{3})x-(3-\sqrt{3})x+[(\sqrt{3}-1)(\sqrt{3}+1)]=0$

$\Rightarrow (\sqrt{3}x)^2-\sqrt{3}(\sqrt{3}+1)x-\sqrt{3}(\sqrt{3}-1)x+[(\sqrt{3}-1)(\sqrt{3}+1)]=0$

$\Rightarrow \sqrt{3}x[\sqrt{3}x-(\sqrt{3}+1)]-(\sqrt{3}-1)[\sqrt{3}x-(\sqrt{3}+1)]=0$

$\Rightarrow [\sqrt{3}x-(\sqrt{3}-1)][\sqrt{3}x-(\sqrt{3}+1)]=0$

$\Rightarrow [\sqrt{3}x-(\sqrt{3}-1)]=0$ and $[\sqrt{3}x-(\sqrt{3}+1)]=0$

$\Rightarrow \sqrt{3}x = (\sqrt{3} - 1)$ and $\sqrt{3}x = (\sqrt{3} + 1)$

$\Rightarrow x = \frac{\sqrt{3}-1}{\sqrt{3}}$ and $x = \frac{\sqrt{3}+1}{\sqrt{3}}$

Rationalizing both values we get,]

$\Rightarrow x = \frac{\sqrt{3}-1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$ and $x = \frac{\sqrt{3}+1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$\Rightarrow x = \frac{\sqrt{3}(\sqrt{3}-1)}{(\sqrt{3})^2}$ and $x = \frac{\sqrt{3}(\sqrt{3}+1)}{(\sqrt{3})^2}$

$\Rightarrow x = \frac{(3-\sqrt{3})}{3}$ and $x = \frac{(3+\sqrt{3})}{3}$

Q47. Solve the following quadratic equations by factorization:

$x - \frac{1}{x - 3} = 3, x \neq 0$

Solution:

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$x - \frac{1}{x - 3} = 3, x \neq 0$

$\Rightarrow x^2 - 3x - 1 = 3x - 9$

$\Rightarrow x^2 - 6x + 8 = 0$

$\Rightarrow x^2 - 4x - 2x + 8 = 0$

$\Rightarrow x(x - 4) - 2(x - 4) = 0$

$\Rightarrow (x - 2)(x - 4) = 0$

$\Rightarrow x = 2,4$

Q48. Solve the following quadratic equations by factorization:

$\frac{1}{x + 4} - \frac{1}{x - 7} = \frac{11}{30}, x \neq 4,7$

Solution:

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$\frac{1}{x + 4} - \frac{1}{x - 7} = \frac{11}{30}, x \neq 4,7$

$\Rightarrow 30(x - 7 - x - 4) = 11(x + 4)(x - 7)$

$\Rightarrow -330 = 11x^2 - 308 - 33x$

$\Rightarrow 11x^2 - 33x + 22 = 0$

$\Rightarrow x^2 - 3x + 2 = 0$

$\Rightarrow x^2 - 2x - x + 2 = 0$

$\Rightarrow x(x - 2) - (x - 2) = 0$

$\Rightarrow (x - 1)(x - 2) = 0$

$\Rightarrow x = 1,2$

Q49. Solve the following quadratic equations by factorization:

$$\frac{1}{x-3}+\frac{2}{x-2}=\frac{8}{x};x\neq 0,2,3$$

Solution:

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$$\frac{1}{x-3}+\frac{2}{x-2}=\frac{8}{x};x\neq 0,2,3$$

$\Rightarrow x((x-2)+2(x-3))=8(x-3)(x-2)$

$\Rightarrow 3x^2-8x=8x^2-40x+48$

$\Rightarrow 5x^2-32x+48=0$

$\Rightarrow 5x^2-20x-12x+48=0$

$\Rightarrow 5x(x-4)-12(x-4)=0$

$\Rightarrow (5x-12)(x-4)=0$

$\Rightarrow x=\frac{12}{5},4$

Q50. Solve the following quadratic equations by factorization:

$$\frac{1}{2a+b+2x}=\frac{1}{2a}+\frac{1}{b}+\frac{1}{2x}$$

Solution:

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$$\frac{1}{2a+b+2x}=\frac{1}{2a}+\frac{1}{b}+\frac{1}{2x}$$

$\Rightarrow \frac{1}{2a+b+2x}-\frac{1}{2x}=\frac{1}{2a}+\frac{1}{b}$

$\Rightarrow 2ab(2x-2a-b-2x)=(2a+b)2x(2a+b+2x)$

$\Rightarrow 2ab(-2a-b)=2(2a+b)(2ax+bx+2x^2)$

$\Rightarrow -ab=2ax+bx+2x^2$

$\Rightarrow 2x^2+2ax+bx+ab=0$

$\Rightarrow 2x(x+a)+b(x+a)=0$

$\Rightarrow (2x+b)(x+a)=0$

$\Rightarrow x=-a,-\frac{b}{2}$

Q51. Solve the following quadratic equations by factorization:

$$\frac{4}{x}-3=\frac{5}{2x+3},x\neq 0,-\frac{3}{2}$$

Solution:

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$\frac{4}{x} - 3 = \frac{5}{2x+3}, x \neq 0, -\frac{3}{2}$

$\Rightarrow (4-3x)(2x+3) = 5x$

$\Rightarrow 8x + 12 - 6x^2 - 9x = 5x$

$\Rightarrow 6x^2 + 6x - 12 = 0$

$\Rightarrow x^2 + x - 2 = 0$

$\Rightarrow x^2 + 2x - x - 2 = 0$

$\Rightarrow x(x+2) - (x+2) = 0$

$\Rightarrow (x-1)(x+2) = 0$

$\Rightarrow x = 1, -2$

Q52. Solve the following quadratic equations by factorization:

$\frac{x-4}{x-5} + \frac{x-6}{x-7} = \frac{10}{3}; x \neq 5,7$

Solution:

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$\frac{x-4}{x-5} + \frac{x-6}{x-7} = \frac{10}{3}; x \neq 5,7$

$\Rightarrow 3(x^2 - 11x + 28) + 3(x^2 - 11x + 30) = 10(x^2 - 12x + 35)$

$\Rightarrow 4x^2 - 54x + 176 = 0$

$\Rightarrow 2x^2 - 27x + 88 = 0$

$\Rightarrow 2x^2 - 16x - 11x + 88 = 0$

$\Rightarrow 2x(x-8) - 11(x-8) = 0$

$\Rightarrow (2x-11)(x-8) = 0$

$\Rightarrow x = \frac{11}{2}, 8$

Q53. Solve the following quadratic equations by factorization:

$\frac{16}{x} - 1 = \frac{15}{x+1}; x \neq 0, -1$

Solution:

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$\frac{16}{x} - 1 = \frac{15}{x+1}; x \neq 0, -1$

$\Rightarrow (16-x)(x+1) = 15x$

$\Rightarrow -x^2 - x + 16x + 16 = 15x$

$\Rightarrow x^2 = 16$
$\Rightarrow x = 4$

Q54. Solve the following quadratic equations by factorization:
$\frac{x-2}{x-3} + \frac{x-4}{x-5} = \frac{10}{3}; x \neq 3,5$

Solution:

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$\frac{x-2}{x-3} + \frac{x-4}{x-5} = \frac{10}{3}; x \neq 3,5$
$\Rightarrow 3(x^2 - 7x + 10) + 3(x^2 - 7x + 12) = 10(x^2 - 8x + 15)$
$\Rightarrow 4x^2 - 38x + 84 = 0$
$\Rightarrow 2x^2 - 19x + 42 = 0$
$\Rightarrow 2x^2 - 12x - 7x + 42 = 0$
$\Rightarrow 2x(x-6) - 7(x-6) = 0$
$\Rightarrow (x-6)(2x-7) = 0$
$\Rightarrow x = \frac{7}{2}, 6$

Q55. Solve the following quadratic equations by factorization:
$\frac{5+x}{5-x} - \frac{5-x}{5+x} = 3\frac{3}{4}; x \neq 5, -5$

Solution:

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$\frac{5+x}{5-x} - \frac{5-x}{5+x} = 3\frac{3}{4}; x \neq 5, -5$
$\Rightarrow 4(25 + x^2 + 10x) - 4(25 + x^2 - 10x) = 15(25 - x^2)$
$\Rightarrow 15x^2 + 80x - 375 = 0$
$\Rightarrow 3x^2 + 16x - 75 = 0$
$\Rightarrow 3x^2 + 25x - 9x - 75 = 0$
$\Rightarrow x(3x + 25) - 3(3x + 25) = 0$
$\Rightarrow (x-3)(3x+25) = 0$
$\Rightarrow x = 3, -\frac{25}{3}.$

Q56. Solve the following quadratic equations by factorization:
$\frac{3}{x+1} - \frac{1}{2} = \frac{2}{3x-1}, x \neq -1, \frac{1}{3}$

Solution:

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$$\frac{3}{x+1}-\frac{1}{2}=\frac{2}{3x-1}, x \neq -1, \frac{1}{3}$$
$$\Rightarrow (5-x)(3x-1)=4x+4$$
$$\Rightarrow -3x^2+16x-5=4x+4$$
$$\Rightarrow 3x^2-12x+9=0$$
$$\Rightarrow 3x^2-3x-9x+9=0$$
$$\Rightarrow 3x(x-1)-9(x-1)=0$$
$$\Rightarrow (3x-9)(x-1)=0$$
$$\Rightarrow x=1,3$$

Q57. Solve the following quadratic equations by factorization:

$$3\left(\frac{3x-1}{2x+3}\right)-2\left(\frac{2x+3}{3x-1}\right)=5; x \neq \frac{1}{3}, -\frac{3}{2}$$

Solution:

Given: $3\left(\frac{3x-1}{2x+3}\right)-2\left(\frac{2x+3}{3x-1}\right)=5; x \neq \frac{1}{3}, -\frac{3}{2}$

to find: Solution of the above quadratic equation.

Solution:In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$$3\left(\frac{3x-1}{2x+3}\right)-2\left(\frac{2x+3}{3x-1}\right)=5; x \neq \frac{1}{3}, -\frac{3}{2}$$
$$\frac{3(3x-1)^2-2(2x+3)^2}{(2x+3)(3x-1)}=5$$
$$\Rightarrow \frac{3(9x^2+1-6x)-2(4x^2+9+12x)}{(6x^2-2x+9x-3)}=5$$
$$\Rightarrow \frac{3(9x^2+1-6x)-2(4x^2+9+12x)}{(6x^2+7x-3)}=5$$
$$\Rightarrow 3(9x^2+1-6x)-2(4x^2+9+12x)=5(6x^2-3+7x)$$
$$\Rightarrow 27x^2+3-18x-8x^2-18-24x=30x^2-15+35x$$
$$\Rightarrow 19x^2-42x-15=30x^2-15+35x$$
$$\Rightarrow 19x^2-30x^2-42x-35x-15+15=0$$
$$\Rightarrow -11x^2-77x=0 \Rightarrow 11x^2+77x=0$$
$$\Rightarrow 11x(x+7)=0$$
$$\Rightarrow x=0 \text{ and } (x+7)=0 \Rightarrow x=0 \text{ and } x=-7$$
$$\Rightarrow x=0,-7$$

Q58. Solve the following quadratic equations by factorization:

$$3\left(\frac{7x+1}{5x-3}\right)-4\left(\frac{5x-3}{7x+1}\right)=11; x \neq \frac{3}{5}, -\frac{1}{7}$$

Solution:

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$3\left(\frac{7x+1}{5x-3}\right)-4\left(\frac{5x-3}{7x+1}\right)=11; x\neq\frac{3}{5},-\frac{1}{7}$

$\Rightarrow 3(7x+1)^2-4(5x-3)^2=11(5x-3)(7x+1)$

$\Rightarrow 3(49x^2+1+14x)-4(25x^2+9-30x)=11(35x^2-3-16x)$

$\Rightarrow 338x^2-338x=0$

$\Rightarrow x(x-1)=0$

$\Rightarrow x=0,1$

Q59. Solve the following quadratic equations by factorization:

$\frac{3}{x+1}+\frac{4}{x-1}=\frac{29}{4x-1}; x\neq 1,-1,\frac{1}{4}$

Solution:

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$\frac{3}{x+1}+\frac{4}{x-1}=\frac{29}{4x-1}; x\neq 1,-1,\frac{1}{4}$

$\Rightarrow (3x-3+4x+4)(4x-1)=29(x^2-1)$

$\Rightarrow (7x+1)(4x-1)=29x^2-29$

$\Rightarrow 28x^2-3x-1=29x^2-29$

$\Rightarrow x^2+3x-28=0$

$\Rightarrow x^2+7x-4x-28=0$

$\Rightarrow x(x+7)-4(x+7)=0$

$\Rightarrow (x-4)(x+7)=0$

$\Rightarrow x=4,-7$

Q60. Solve the following quadratic equations by factorization:

$\frac{2}{x+1}+\frac{3}{2(x-2)}=\frac{23}{5x}; x\neq 0,-1,2$

Solution:

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized

$\frac{2}{x+1}+\frac{3}{2(x-2)}=\frac{23}{5x}; x\neq 0,-1,2$

$\Rightarrow 5x(4x-8+3x+3)=46(x+1)(x-2)$

$\Rightarrow 35x^2-25x=46x^2-92-46x$

$\Rightarrow 11x^2-19x-92=0$

$\Rightarrow 11x^2 - 44x + 23x - 92 = 0$
$\Rightarrow 11x(x-4) + 23(x-4) = 0$
$\Rightarrow (11x+23)(x-4) = 0$
$\Rightarrow x = 4, -\frac{23}{11}$

Exercise 8.4

Q1. Find the roots of the following quadratic (if they exist) by the method of completing the square.
$x^2 - 4\sqrt{2}x + 6 = 0$
Solution:
Given: $x^2 - 4\sqrt{2}x + 6 = 0$
To find: the roots of the following quadratic (if they exist) by the method of completing the square.
We have to make the quadratic equation a perfect square if possible or sum of perfect square with a constant.
Step 1: Make the coefficient of x^2 unity. In the equation $x^2 - 4\sqrt{2}x + 6 = 0$, The coefficient of x^2 is 1.
Step 2: Shift the constant term on RHS,
$\Rightarrow x^2 - 4\sqrt{2}x = -6$
Step 3: Add square of half of coefficient of x on both the sides.
$\Rightarrow x^2 - 2 \times 2\sqrt{2}x + (2\sqrt{2})^2 = -6 + (2\sqrt{2})^2$
Step 4: Apply the formula, $(a-b)^2 = a^2 - 2ab + b^2$ on LHS and solve RHS,
Here $a = x$ and $b = 2\sqrt{2}$
$\Rightarrow (x - 2\sqrt{2})^2 = -6 + (2\sqrt{2})^2$
$\Rightarrow (x - 2\sqrt{2})^2 = -6 + (2)^2(\sqrt{2})^2$
$\Rightarrow (x - 2\sqrt{2})^2 = -6 + (4 \times 2)$
$\Rightarrow (x - 2\sqrt{2})^2 = -6 + 8$
$\Rightarrow (x - 2\sqrt{2})^2 = 2$
As RHS is positive, the roots exist.
Now, take square root on both sides,
$\Rightarrow (x - 2\sqrt{2}) = \pm\sqrt{2}$
$\Rightarrow x = \sqrt{2} + 2\sqrt{2}$ and $x = -\sqrt{2} + 2\sqrt{2}$
$\Rightarrow x = 3\sqrt{2}$ and $x = \sqrt{2}$

Q2. Find the roots of the following quadratic (if they exist) by the method of completing the square.
$2x^2 - 7x + 3 = 0$
Solution:
We have to make the quadratic equation a perfect square if possible or sum of perfect square with a constant.

$2x^2 - 7x + 3 = 0$ Divide the equation by 2 to get,

$x^2 - \frac{7}{2}x + \frac{3}{2} = 0$

Now add and subtract the square of half of coefficient of x to get,

$\Rightarrow x^2 - 2 \times \frac{7}{4}x + \left(\frac{7}{4}\right)^2 - \left(\frac{7}{4}\right)^2 + \frac{3}{2} = 0$

Use the formula $(a + b)^2 = a^2 + 2ab + b^2$

$\Rightarrow \left(x - \frac{7}{4}\right)^2 - \frac{49}{16} + \frac{3}{2} = 0$

$\Rightarrow \left(x - \frac{7}{4}\right)^2 - \frac{25}{16} = 0$

$\Rightarrow \left(x - \frac{7}{4}\right)^2 = \frac{25}{16}$

As RHS is positive the roots exist.

$\Rightarrow x - \frac{7}{4} = \pm\frac{5}{4}$

$\Rightarrow x - \frac{7}{4} = \frac{5}{4}$ and $x - \frac{7}{4} = -\frac{5}{4}$

$\Rightarrow x = \frac{7}{4} + \frac{5}{4}$ and $x = \frac{7}{4} - \frac{5}{4}$

$\Rightarrow x = \frac{12}{4}$ and $x = \frac{2}{4}$

$\Rightarrow x = 3, \frac{1}{2}$

Q3. Find the roots of the following quadratic (if they exist) by the method of completing the square.

$3x^2 + 11x + 10 = 0$

Solution:

We have to make the quadratic equation a perfect square if possible or sum of perfect square with a constant.

$(a + b)^2 = a^2 + 2ab + b^2$

$3x^2 + 11x + 10 = 0$

$\Rightarrow x^2 + 2 \times \frac{11}{6}x + \left(\frac{11}{6}\right)^2 - \left(\frac{11}{6}\right)^2 + \frac{10}{3} = 0$

$\Rightarrow \left(x + \frac{11}{6}\right)^2 + \frac{10}{3} - \frac{121}{36} = 0$

$\Rightarrow \left(x + \frac{11}{6}\right)^2 = \frac{1}{36}$

$\Rightarrow x + \frac{11}{6} = \frac{1}{6}$

$\Rightarrow x = -2, -\frac{5}{3}$

Q4. Find the roots of the following quadratic (if they exist) by the method of completing the square.

$2x^2 + x - 4 = 0$

Solution:

We have to make the quadratic equation a perfect square if possible or sum of perfect square with a constant.

$(a+b)^2 = a^2 + 2ab + b^2$

$2x^2 + x - 4 = 0$

$\Rightarrow x^2 + \frac{x}{2} - 2 = 0$

$\Rightarrow x^2 + 2 \times \frac{1}{4} \times x + \left(\frac{1}{4}\right)^2 - \left(\frac{1}{4}\right)^2 - 2 = 0$

$\Rightarrow \left(x + \frac{1}{4}\right)^2 = \frac{33}{16}$

$\Rightarrow x + \frac{1}{4} = \frac{\sqrt{33}}{4}$

$\Rightarrow x = \frac{\sqrt{33} - 1}{4}, \frac{-\sqrt{33} - 1}{4}$

Q5. Find the roots of the following quadratic (if they exist) by the method of completing the square.

$2x^2 + x + 4 = 0$

Solution:

We have to make the quadratic equation a perfect square if possible or sum of perfect square with a constant.

$(a+b)^2 = a^2 + 2ab + b^2$

$2x^2 + x + 4 = 0$

$\Rightarrow x^2 + \frac{x}{2} + 2 = 0$

$\Rightarrow x^2 + 2 \times \frac{1}{4} \times x + \left(\frac{1}{4}\right)^2 - \left(\frac{1}{4}\right)^2 + 2 = 0$

$\Rightarrow \left(x + \frac{1}{4}\right)^2 = -\frac{31}{16}$

Roots are not real.

Q6. Find the roots of the following quadratic (if they exist) by the method of completing the square.

$4x^2 + 4\sqrt{3}x + 3 = 0$

Solution:

Given: The quadratic equation $4x^2 + 4\sqrt{3}x + 3 = 0$

To find: the roots of the following quadratic (if they exist) by the method of completing the square.

We have to make the quadratic equation a perfect square if possible or sum of

perfect square with a constant. Step 1: Make the coefficient of x^2 unity. In the equation $4x^2 + 4\sqrt{3}x + 3 = 0$, The coefficient of x^2 is 4. So to make the coefficient of x^2 equals to 1. divide the whole equation by 4.
The quadratic equation now becomes:

$\frac{4x^2}{4} + \frac{4\sqrt{3}x}{4} + \frac{3}{4} = \frac{0}{4}$

$\Rightarrow x^2 + \sqrt{3}x + \frac{3}{4} = 0$

Step 2: Shift the constant term on RHS,

$\Rightarrow x^2 + \sqrt{3}x = -\frac{3}{4}$

Step 3: Add square of half of coefficient of x on both the sides.

$\Rightarrow x^2 + \sqrt{3}x + \left(\frac{\sqrt{3}}{2}\right)^2 = -\frac{3}{4} + \left(\frac{\sqrt{3}}{2}\right)^2$

Step 4: Apply the formula, $(a + b)^2 = a^2 + 2ab + b^2$ on LHS and solve RHS,

Here $a = x$ and $b = \frac{\sqrt{3}}{2}$

$$\Rightarrow \left(x + \frac{\sqrt{3}}{2}\right)^2 = -\frac{3}{4} + \frac{(\sqrt{3})^2}{(2)^2}$$

$$\Rightarrow \left(x + \frac{\sqrt{3}}{2}\right)^2 = -\frac{3}{4} + \frac{3}{4}$$

$$\Rightarrow \left(x + \frac{\sqrt{3}}{2}\right)^2 = 0$$

Step 5: As the RHS is zero, the roots exist. Since the quadratic equations have 2 roots, in this case both roots will be same.

$\Rightarrow x = -\frac{\sqrt{3}}{2}, -\frac{\sqrt{3}}{2}$

Q7. Find the roots of the following quadratic (if they exist) by the method of completing the square.

Solution:

We have to make the quadratic equation a perfect square if possible or sum of perfect square with a constant.

$(a + b)^2 = a^2 + 2ab + b^2$

$\sqrt{2}x^2 - 3x - 2\sqrt{2} = 0$

$$\Rightarrow x^2 - 2 \times \left(\frac{3}{2\sqrt{2}}\right) \times x + \left(\frac{3}{2\sqrt{2}}\right)^2 - \left(\frac{3}{2\sqrt{2}}\right)^2 - 2 = 0$$

$$\Rightarrow \left(x - \frac{3}{2\sqrt{2}}\right)^2 = 2 + \frac{9}{8} = \frac{25}{8}$$

$$\Rightarrow x - \frac{3}{2\sqrt{2}} = \frac{5}{2\sqrt{2}}$$

$$\Rightarrow x = 2\sqrt{2}, -\frac{1}{\sqrt{2}}$$

Q8. Find the roots of the following quadratic (if they exist) by the method of completing the square.

$\sqrt{3}x^2 + 10x + 7\sqrt{3} = 0$

Solution:

Given: $\sqrt{3}x^2 + 10x + 7\sqrt{3} = 0$

To find: the roots of the following quadratic (if they exist) by the method of completing the square.

Solution: We have to make the quadratic equation a perfect square if possible or sum of perfect square with a constant.

Step 1: Make the coefficient of x^2 unity. In the equation

$\sqrt{3}x^2 + 10x + 7\sqrt{3} = 0,$

The coefficient of x^2 is $\sqrt{3}$.

So to make the coffecient of x^2 equals to 1 .

divide the whole equation by $\sqrt{3}$.

The quadratic equation now becomes:

$$\frac{\sqrt{3}x^2}{\sqrt{3}} + \frac{10x}{\sqrt{3}} + \frac{7\sqrt{3}}{\sqrt{3}} = \frac{0}{\sqrt{3}}$$

$$\Rightarrow x^2 + \frac{10x}{\sqrt{3}} + 7 = 0$$

Step 2: Shift the constant term on RHS,

$x^2 + \frac{10}{\sqrt{3}} = -7$

Step 3: Add square of half of coefficient of x on both sides.

$\Rightarrow x^2 + \frac{10x}{\sqrt{3}} + \left(\frac{5}{\sqrt{3}}\right)^2 = -7 + \left(\frac{5}{\sqrt{3}}\right)^2$

Step 4: Apply the formula, $(a+b)^2 = a^2 + 2ab + b^2$ on LHS and solve RHS.

Here $a = x$ and $b = \frac{5}{\sqrt{3}}$

$$\Rightarrow \left(x + \frac{5}{\sqrt{3}}\right)^2 = -7 + \frac{(5)^2}{(\sqrt{3})^2}$$

$$\Rightarrow \left(x + \frac{5}{\sqrt{3}}\right)^2 = -7 + \frac{25}{3}$$

$$\Rightarrow \left(x + \frac{5}{\sqrt{3}}\right)^2 = \frac{-21 + 25}{3}$$

$$\Rightarrow \left(x + \frac{5}{\sqrt{3}}\right)^2 = \frac{4}{3}$$

As RHS is positive, the roots exist.

Step 5: take square root on both sides,

$$\Rightarrow \left(x + \frac{5}{\sqrt{3}}\right) = \pm\frac{\sqrt{4}}{\sqrt{3}}$$

$$\Rightarrow \left(x+\frac{5}{\sqrt{3}}\right)=\pm\frac{2}{\sqrt{3}}$$

$$\Rightarrow x+\frac{5}{\sqrt{3}}=\frac{2}{\sqrt{3}} \text{ and } x+\frac{5}{\sqrt{3}}=-\frac{2}{\sqrt{3}}$$

$$\Rightarrow x=\frac{2}{\sqrt{3}}-\frac{5}{\sqrt{3}} \text{ and } x=-\frac{2}{\sqrt{3}}-\frac{5}{\sqrt{3}}$$

$$\Rightarrow x=\frac{-3}{\sqrt{3}} \text{ and } x=\frac{-7}{\sqrt{3}}$$

$$\Rightarrow x=-\sqrt{3} \text{ and } x=-\frac{7}{\sqrt{3}}$$

Q9. Find the roots of the following quadratic (if they exist) by the method of completing the square.

$x^2-(\sqrt{2}+1)x+\sqrt{2}=0$

Solution:

We have to make the quadratic equation a perfect square if possible or sum of perfect square with a constant.

$(a+b)^2=a^2+2ab+b^2$

$x^2-(\sqrt{2}+1)x+\sqrt{2}=0$

$$\Rightarrow x^2-(\sqrt{2}+1)x+\left(\frac{\sqrt{2}+1}{2}\right)^2-\left(\frac{\sqrt{2}+1}{2}\right)^2+\sqrt{2}=0$$

$$\Rightarrow \left(x-\frac{\sqrt{2}+1}{2}\right)^2=\frac{2+1+2\sqrt{2}}{4}-\sqrt{2}$$

$$\Rightarrow \left(x-\frac{\sqrt{2}+1}{2}\right)^2=\frac{2+1-2\sqrt{2}}{4}=\left(\frac{\sqrt{2}-1}{2}\right)^2$$

$$\Rightarrow x-\frac{\sqrt{2}+1}{2}=\frac{\sqrt{2}-1}{2}$$

$\Rightarrow x=\sqrt{2},1$

Q10. Find the roots of the following quadratic (if they exist) by the method of completing the square.

$x^2-4ax+4a^2-b^2=0$

Solution:

We have to make the quadratic equation a perfect square if possible or sum of perfect square with a constant.

$(a+b)^2=a^2+2ab+b^2$

$x^2-4ax+4a^2-b^2=0$

$\Rightarrow x^2-2\times 2ax+4a^2=b^2$

$\Rightarrow (x-2a)^2=b^2$

$\Rightarrow x-2a=\pm b$

$\Rightarrow x=2a+b,2a-b$

Exercise 8.5

Q1. Write the discriminant of the following quadratic equations:

(i) $2x^2 - 5x + 3 = 0$

(ii) $x^2 + 2x + 4 = 0$

(iii) $(x - 1)(2x - 1) = 0$

(iv) $x^2 - 2x + k = 0, k \in R$

(v) $\sqrt{3}x^2 + 2\sqrt{2}x - 2\sqrt{3} = 0$

(vi) $x^2 - x + 1 = 0$

Solution:

(i) $2x^2 - 5x + 3 = 0$

For a quadratic equation, $ax^2 + bx + c = 0$,

Discriminant, $D = b^2 - 4ac$

$2x^2 - 5x + 3 = 0$

$\Rightarrow D = 25 - 4 \times 2 \times 3 = 1$

(ii) $x^2 + 2x + 4 = 0$

For a quadratic equation, $ax^2 + bx + c = 0$,

Discriminant, $D = b^2 - 4ac$

Given, $x^2 + 2x + 4 = 0$

$\Rightarrow D = 4 - 4 \times 4 \times 1 = -12$

(iii) $(x - 1)(2x - 1) = 0$

For a quadratic equation, $ax^2 + bx + c = 0$,

Discriminant, $D = b^2 - 4ac$

Given, $(x - 1)(2x - 1) = 0$

$\Rightarrow 2x^2 - 3x + 1 = 0$

$\Rightarrow D = 9 - 4 \times 2 \times 1 = 1$

(iv) $x^2 - 2x + k = 0, k \in R$

For a quadratic equation, $ax^2 + bx + c = 0$,

Discriminant, $D = b^2 - 4ac$

$x^2 - 2x + k = 0, k \in R$

$\Rightarrow D = 4 - 4 \times 1 \times k = 4 - 4k$

(v) $\sqrt{3}x^2 + 2\sqrt{2}x - 2\sqrt{3} = 0$

For a quadratic equation, $ax^2 + bx + c = 0$,

Discriminant, $D = b^2 - 4ac$

$\sqrt{3}x^2 + 2\sqrt{2}x - 2\sqrt{3} = 0$

$\Rightarrow D = 8 - 4 \times \sqrt{3} \times -2\sqrt{3} = 32$

(vi) $x^2 - x + 1 = 0$
For a quadratic equation, $ax^2 + bx + c = 0$,
Discriminant, $D = b^2 - 4ac$
Given, $x^2 - x + 1 = 0$
$\Rightarrow D = 1 - 4 \times 1 = -3$

Q2. In the following determine whether the given quadratic equations have real roots and if so, find the roots:
(i) $16x^2 = 24x + 1$
(ii) $x^2 + x + 2 = 0$
(iii) $\sqrt{3}x^2 + 10x - 8\sqrt{3} = 0$
(iv) $3x^2 - 2x + 2 = 0$
(v) $2x^2 - 2\sqrt{6}x + 3 = 0$
(vi) $3a^2x^2 + 8abx + 4b^2 = 0, a \neq 0$
(vii) $3x^2 + 2\sqrt{5}x - 5 = 0$
(viii) $x^2 - 2x + 1 = 0$
(ix) $2x^2 + 5\sqrt{3}x + 6 = 0$
(x) $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$
(xi) $2x^2 - 2\sqrt{2}x + 1 = 0$
(xii) $3x^2 - 5x + 2 = 0$

Solution:

(i) $16x^2 = 24x + 1$
For a quadratic equation, $ax^2 + bx + c = 0$,
$D = b^2 - 4ac$
If $D < 0$, roots are not real
If $D > 0$, roots are real and unequal
If $D = 0$, roots are real and equal
$16x^2 - 24x - 1 = 0$
$\Rightarrow D = 24 \times 24 + 4 \times 16 \times 1 = 640$
Roots are real.

$$x = \frac{24 \pm \sqrt{576 + 4 \times 16}}{32}$$

$$\Rightarrow x = \frac{3 \pm \sqrt{10}}{4}$$

(ii) $x^2 + x + 2 = 0$
For a quadratic equation, $ax^2 + bx + c = 0$,
$D = b^2 - 4ac$
If $D < 0$, roots are not real

If $D > 0$, roots are real and unequal
If $D = 0$, roots are real and equal
$x^2 + x + 2 = 0$
$\Rightarrow D = 1 - 4 \times 2 = -7$
Roots are not real

(iii) $\sqrt{3}x^2 + 10x - 8\sqrt{3} = 0$
For a quadratic equation, $ax^2 + bx + c = 0$,
$D = b^2 - 4ac$
If $D < 0$, roots are not real
If $D > 0$, roots are real and unequal
If $D = 0$, roots are real and equal
$\sqrt{3}x^2 + 10x - 8\sqrt{3} = 0$
$\Rightarrow D = 100 + 4 \times 8\sqrt{3} \times \sqrt{3} = 196$
Roots are real

$$x = \frac{-10 \pm \sqrt{100 + 4 \times 8\sqrt{3} \times \sqrt{3}}}{2\sqrt{3}}$$

$$\Rightarrow x = \frac{-10 > 14}{2\sqrt{3}}$$

$\Rightarrow x = -4\sqrt{3}, \frac{2}{\sqrt{3}}$

(iv) $3x^2 - 2x + 2 = 0$
For a quadratic equation, $ax^2 + bx + c = 0$,
$D = b^2 - 4ac$
If $D < 0$, roots are not real
If $D > 0$, roots are real and unequal
If $D = 0$, roots are real and equal
$3x^2 - 2x + 2 = 0$
$\Rightarrow \mathrm{D} = 4 - 4 \times 2 \times 3 = -20$
Roots are not real

(v) $2x^2 - 2\sqrt{6}x + 3 = 0$
For a quadratic equation, $ax^2 + bx + c = 0$,
$D = b^2 - 4ac$
If $\mathrm{D} < 0$, roots are not real
If $D > 0$, roots are real and unequal
If $D = 0$, roots are real and equal
$2x^2 - 2\sqrt{6}x + 3 = 0$
$\Rightarrow D = 4 \times 6 - 4 \times 3 \times 2 = 0$
Roots are equal

$$x=\frac{2\sqrt{6}\pm\sqrt{24-4\times3\times2}}{4}=\sqrt{\frac{3}{2}}$$

(vi) $3a^2x^2+8abx+4b^2=0, a\neq0$
For a quadratic equation, $ax^2+bx+c=0$,
$D=b^2-4ac$
If $D<0$, roots are not real
If $D>0$, roots are real and unequal
If $D=0$, roots are real and equal
$3a^2x^2+8abx+4b^2=0, a\neq0$
$\Rightarrow D=64a^2b^2-4\times3a^2\times4b^2=16a^2b^2$

$$x=\frac{-8ab\pm\sqrt{64a^2b^2-48a^2b^2}}{6a^2}$$

$$\Rightarrow x=\frac{-8ab\pm4ab}{6a^2}=-\frac{2b}{a},-\frac{2b}{3a}$$

(vii) $3x^2+2\sqrt{5}x-5=0$
For a quadratic equation, $ax^2+bx+c=0$,
$D=b^2-4ac$
If $D<0$, roots are not real
If $D>0$, roots are real and unequal
If $D=0$, roots are real and equal
$3x^2+2\sqrt{5}x-5=0$
$D=20+4\times5\times3=80$

$$\Rightarrow x=\frac{-2\sqrt{5}\pm\sqrt{20-4\times3\times-5}}{6}$$

$$\Rightarrow x=\frac{-2\sqrt{5}\pm4\sqrt{5}}{6}$$

$$\Rightarrow x=-\sqrt{5},\sqrt{\frac{5}{3}}$$

(viii) $x^2-2x+1=0$
For a quadratic equation, $ax^2+bx+c=0$,
$D=b^2-4ac$
If $D<0$, roots are not real
If $D>0$, roots are real and unequal
If $D=0$, roots are real and equal
$x^2-2x+1=0$
$\Rightarrow D=4-4\times1\times1=0$
Roots are equal

$$x = \frac{2 \pm \sqrt{(4-4)}}{2} = 1$$

(ix) $2x^2 + 5\sqrt{3}x + 6 = 0$

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If $D < 0$, roots are not real

If $D > 0$, roots are real and unequal

If $D = 0$, roots are real and equal

$2x^2 + 5\sqrt{3}x + 6 = 0$

$\Rightarrow D = 75 - 4 \times 2 \times 6 = 27$

$x = \frac{-5\sqrt{3} \pm \sqrt{27}}{4}$

$\Rightarrow x = \frac{-5\sqrt{3} \pm 3\sqrt{3}}{4}$

$\Rightarrow x = -2\sqrt{3}, -\sqrt{\frac{3}{2}}$

(x) $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If $D < 0$, roots are not real

If $D > 0$, roots are real and unequal

If $D = 0$, roots are real and equal

$\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

$\Rightarrow \mathrm{D} = 49 - 4 \times 5\sqrt{2} \times \sqrt{2} = 9$

$$x = \frac{-7 \pm \sqrt{9}}{2\sqrt{2}} = \frac{-7 \pm 3}{2\sqrt{2}}$$

$$\Rightarrow x = -\frac{5}{\sqrt{2}}, -\sqrt{2}$$

(xi) $2x^2 - 2\sqrt{2}x + 1 = 0$

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If $D < 0$, roots are not real

If $D > 0$, roots are real and unequal

If $D = 0$, roots are real and equal

$2x^2 - 2\sqrt{2}x + 1 = 0$

$\mathrm{D} = (2\sqrt{2})^2 - 4 \times 2 \times 1$

$\Rightarrow D = 8 - 8 = 0$

Roots are equal

$$x = \frac{2\sqrt{2}}{4}$$

$$\Rightarrow x = \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$$

(xii) $3x^2 - 5x + 2 = 0$

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If $D < 0$, roots are not real

If $D > 0$, roots are real and unequal

If $D = 0$, roots are real and equal

$3x^2 - 5x + 2 = 0$

$\Rightarrow D = 25 - 4 \times 3 \times 2 = 1$

$x = \frac{(5)\sqrt{1}}{6}$

$\Rightarrow x = \frac{5 \times 1}{6}$

$\Rightarrow x = 1, \frac{2}{3}$

Q3. Solve for x:

(i) $\frac{x-1}{x-2} + \frac{x-3}{x-4} = 3\frac{1}{3}; x \neq 2,4$

(ii) $\frac{1}{x} - \frac{1}{x-2} = 3, x \neq 0,2$

(iii) $x + \frac{1}{x} = 3, x \neq 0$

(iv) $\frac{16}{x} - 1 = \frac{15}{x+1}, x \neq 0, -1$

Solution:

(i) $\frac{x-1}{x-2} + \frac{x-3}{x-4} = 3\frac{1}{3}; x \neq 2,4$

$$\frac{x-1}{x-2} + \frac{x-3}{x-4} = \frac{10}{3}$$

$$3\left(\frac{x-1}{x-2} + \frac{x-3}{x-4}\right) = 10$$

$$3\left(\frac{x-1}{x-2}\right) + 3\left(\frac{x-3}{x-4}\right) = 10$$

LCM of the denominator is $(x-2)(x-4)$ Now further solve it,

$\Rightarrow 3(x-1)(x-4) + 3(x-2)(x-3) = 10(x-2)(x-4)$

$\Rightarrow 3(x^2 - 5x + 4) + 3(x^2 - 6x + 6) = 10(x^2 - 6x + 8)$

$\Rightarrow 3(x^2 - 4x - x + 4) + 3(x^2 - 3x - 2x + 6) = 10(x^2 - 4x - 2x + 8)$

$\Rightarrow 3x^2 + 12 - 15x + 3x^2 + 18 - 18x = 10x^2 - 60x + 80$

$\Rightarrow 6x^2 - 30x + 30 = 10x^2 - 60x + 80 \Rightarrow 6x^2 - 10x^2 - 30x + 60x + 30 - 80 = 0$

$\Rightarrow -4x^2 + 30x - 50 = 0$

$\Rightarrow 4x^2 - 30x + 50 = 0$

$\Rightarrow 2x^2 - 15x + 25 = 0$

Factorize it by splitting the middle term.

$\Rightarrow 2x^2 - 10x - 5x + 25 = 0$

$\Rightarrow 2x(x-5) - 5(x-5) = 0$

$\Rightarrow (2x-5)(x-5) = 0 \Rightarrow 2x - 5 = 0\ x = \frac{5}{2}$

$\Rightarrow x - 5 = 0\ x = 5$

$Thus, x = 5, \frac{5}{2}$

(ii) $\frac{1}{x} - \frac{1}{x-2} = 3, x \neq 0,2$

LCM of the denominator is $x(x-2)$

$\Rightarrow \frac{x-2-x}{x(x-2)} = 3$

$\Rightarrow x - 2 - x = 3x(x-2)$

$\Rightarrow x - 2 - x = 3x^2 - 6x$

$\Rightarrow 3x^2 - 6x + 2 = 0$

Using $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

Here $a = 3, b = -6, c = 2$

$\Rightarrow x = \frac{6 \pm \sqrt{36-24}}{6} = \frac{6 \pm \sqrt{12}}{6}$

$\Rightarrow x = \frac{3 \pm \sqrt{3}}{3}$

(iii) $x + \frac{1}{x} = 3, x \neq 0$

LCM of denominators is x .

$\Rightarrow \frac{x^2+1}{x} = 3$

$\Rightarrow x^2 + 1 = 3x$

$\Rightarrow x^2 - 3x + 1 = 0$

Using $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{3 \pm \sqrt{3^2-4}}{2}$

$x = \frac{3 \pm \sqrt{9-4}}{2}$

$x = \frac{3 \pm \sqrt{5}}{2}$

(iv) $\frac{16}{x} - 1 = \frac{15}{x+1}, x \neq 0, -1$

$\frac{16-x}{x} = \frac{15}{x+1}$

Use cross multiplication to get, $(16-x)(x+1) = 15x$

$\Rightarrow 16x - x^2 + 16 - x = 15x$
$\Rightarrow 16x - 15x - x^2 + 16 - x = 0 \Rightarrow -x^2 + 16 = 0$
$\Rightarrow x^2 = 16$
$\Rightarrow x = \pm 4$

Exercise 8.6

Q1. Determine the nature of the roots of the following quadratic equations:
(i) $2x^2 - 3x + 5 = 0$
(ii) $2x^2 - 6x + 3 = 0$
(iii) $\frac{3}{5}x^2 - \frac{2}{3}x + 1 = 0$
(iv) $3x^2 - 4\sqrt{3}x + 4 = 0$
(v) $3x^2 - 2\sqrt{6}x + 2 = 0$
(vi) $(x - 2a)(x - 2b) = 4ab$
(vii) $9a^2b^2x^2 - 24abcdx + 16c^2d^2 = 0, a \neq 0, b \neq 0$
(viii) $2(a^2 + b^2)x^2 + 2(a + b)x + 1 = 0$
(ix) $(b + c)x^2 - (a + b + c)x + a = 0$

Solution:
(i) $2x^2 - 3x + 5 = 0$
For a quadratic equation, $ax^2 + bx + c = 0$,
$D = b^2 - 4ac$
If $D < 0$, roots are not real
If $D > 0$, roots are real and unequal
If $D = 0$, roots are real and equal
$2x^2 - 3x + 5 = 0$
$\Rightarrow D = 9 - 4 \times 5 \times 2 = -31$
Roots are not real.

(ii) $2x^2 - 6x + 3 = 0$
For a quadratic equation, $ax^2 + bx + c = 0$,
$D = b^2 - 4ac$
If $D < 0$, roots are not real
If $D > 0$, roots are real and unequal
If $D = 0$, roots are real and equal
$2x^2 - 6x + 3 = 0$
$\Rightarrow D = 36 - 4 \times 2 \times 3 = 12$
Roots are real and distinct.

(iii) $\frac{3}{5}x^2 - \frac{2}{3}x + 1 = 0$

For a quadratic equation, $ax^2 + bx + c = 0$,
$D = b^2 - 4ac$
If $D < 0$, roots are not real
If $D > 0$, roots are real and unequal
If $D = 0$, roots are real and equal
$\frac{3}{5}x^2 - \frac{2}{3}x + 1 = 0$
$\Rightarrow D = \frac{4}{9} - 4 \times \frac{3}{5} \times 1 = -\frac{88}{45}$
Roots are not real.

(iv) $3x^2 - 4\sqrt{3}x + 4 = 0$
For a quadratic equation, $ax^2 + bx + c = 0$,
$D = b^2 - 4ac$
If $D < 0$, roots are not real
If $D > 0$, roots are real and unequal
If $D = 0$, roots are real and equal
$3x^2 - 4\sqrt{3}x + 4 = 0$
$\Rightarrow D = 48 - 4 \times 3 \times 4 = 0$
Roots are real and equal

(v) $3x^2 - 2\sqrt{6}x + 2 = 0$
For a quadratic equation, $ax^2 + bx + c = 0$,
$D = b^2 - 4ac$
If $D < 0$, roots are not real
If $D > 0$, roots are real and unequal
If $D = 0$, roots are real and equal
$3x^2 - 2\sqrt{6}x + 2 = 0$
$\Rightarrow D = 24 - 4 \times 3 \times 2 = 0$
Roots are real and equal.

(vi) $(x - 2a)(x - 2b) = 4ab$
For a quadratic equation, $ax^2 + bx + c = 0$,
$D = b^2 - 4ac$
If $D < 0$, roots are not real
If $D > 0$, roots are real and unequal
If $D = 0$, roots are real and equal
$(x - 2a)(x - 2b) = 4ab$
$\Rightarrow x^2 - (2a + 2b)x + 4ab = 4ab$
$\Rightarrow x^2 - (2a + 2b)x = 0$
$D = (2a + 2b)^2 - 0 = (2a + 2b)^2$
Roots are real and distinct

(vii) $9a^2b^2x^2 - 24abcdx + 16c^2d^2 = 0, a \neq 0, b \neq 0$
For a quadratic equation, $ax^2 + bx + c = 0$,
$D = b^2 - 4ac$
If $D < 0$, roots are not real
If $D > 0$, roots are real and unequal
If $D = 0$, roots are real and equal
$9a^2b^2x^2 - 24abcdx + 16c^2d^2 = 0$
$\Rightarrow D = 576a^2b^2c^2d^2 - 4 \times 16 \times 9 \times a^2b^2c^2d^2 = 0$
Roots are real and equal

(viii) $2(a^2 + b^2)x^2 + 2(a + b)x + 1 = 0$
For a quadratic equation, $ax^2 + bx + c = 0$,
$D = b^2 - 4ac$
If $D < 0$, roots are not real
If $D > 0$, roots are real and unequal
If $D = 0$, roots are real and equal
$2(a^2 + b^2)x^2 + 2(a + b)x + 1 = 0$
$\Rightarrow D = 4(a + b)^2 - 4 \times 2 \times (a^2 + b^2)$
$\Rightarrow D = -4(a^2 + b^2) + 2ab = -(a - b)^2 - 3(a^2 + b^2)$
Roots are not real

(ix) $(b + c)x^2 - (a + b + c)x + a = 0$
For a quadratic equation, $ax^2 + bx + c = 0$,
$D = b^2 - 4ac$
If $D < 0$, roots are not real
If $D > 0$, roots are real and unequal
If $D = 0$, roots are real and equal
$(b + c)x^2 - (a + b + c)x + a = 0$
$\Rightarrow D = (a + b + c)^2 - 4a(b + c)$
$\Rightarrow D = a^2 + b^2 + c^2 - 2ab - 2ac + 2bc$
$\Rightarrow D = (a - b - c)^2$
Thus, roots are real and unequal.

Q2. Find the values of k for which the roots are real and equal in each of the following equations:
(i) $kx^2 + 4x + 1 = 0$
(ii) $kx^2 - 2\sqrt{5}x + 4 = 0$
(iii) $3x^2 - 5x + 2k = 0$
(iv) $4x^2 + kx + 9 = 0$
(v) $2kx^2 - 40x + 25 = 0$
(vi) $9x^2 - 24x + k = 0$

(vii) $4x^2 - 3kx + 1 = 0$
(viii) $x^2 - 2(5 + 2k)x + 3(7 + 10k) = 0$
(ix) $(3k + 1)x^2 + 2(k + 1)x + k = 0$
(x)$kx^2 + kx + 1 = -4x^2 - x$
(xi) $(k + 1)x^2 + 2(k + 3)x + (k + 8) = 0$
(xii) $x^2 - 2kx + 7x + 1/4 = 0$
(xiii) $(k + 1)x^2 - 2(3k + 1)x + 8k + 1 = 0$
(xiv) $5x^2 - 4x + 2 + k(4x^2 - 2x - 1) = 0$
(xv) $(4 - k)x^2 + (2k + 4)x + (8k + 1) = 0$
(xvi) $(2k + 1)x^2 + 2(k + 3)x + (k + 5) = 0$
(xvii) $4x^2 - 2(k + 1)x + (k + 4) = 0$
(xviii) $x^2 - 2(k + 1)x + k^2 = 0$
(xix) $k^2x^2 - 2(k - 1)x + 4 = 0$
$(xx)(k + 1)x^2 - 2(k - 1)x + 1 = 0$
(xxi) $2x^2 + kx + 3 = 0$
(xxii) $kx(x - 2) + 6 = 0$
(xxiii) $x^2 - 4kx + k = 0$
(xxiv) $kx(x - 2\sqrt{5}) + 10 = 0$
$(xxv)px(x - 3) + 9 = 0$
$(xxvi)4x^2 + px + 3 = 0$

Solution:

(i) $kx^2 + 4x + 1 = 0$
For a quadratic equation, $ax^2 + bx + c = 0$,
$D = b^2 - 4ac$
If $D = 0$, roots are real and equal
$kx^2 + 4x + 1 = 0$
$\Rightarrow D = 16 - 4k = 0$
$\Rightarrow k = 4$

(ii) $kx^2 - 2\sqrt{5}x + 4 = 0$
For a quadratic equation, $ax^2 + bx + c = 0$,
$D = b^2 - 4ac$
If $D = 0$, roots are real and equal
$kx^2 - 2\sqrt{5}x + 4 = 0$
$\Rightarrow D = 4 \times 5 - 4 \times 4k = 0$
$\Rightarrow k = \frac{5}{4}$

(iii) $3x^2 - 5x + 2k = 0$
For a quadratic equation, $ax^2 + bx + c = 0$,
$D = b^2 - 4ac$
If $D = 0$, roots are real and equal

$3x^2 - 5x + 2k = 0$
$\Rightarrow D = 25 - 4 \times 3 \times 2k = 0$
$\Rightarrow k = \dfrac{25}{24}$

(iv) $4x^2 + kx + 9 = 0$
For a quadratic equation, $ax^2 + bx + c = 0$,
$D = b^2 - 4ac$
If $D = 0$, roots are real and equal
$4x^2 + kx + 9 = 0$
$\Rightarrow D = k^2 - 4 \times 4 \times 9 = 0$
$\Rightarrow k^2 - 144 = 0$
$\Rightarrow k = 12$

(v) $2kx^2 - 40x + 25 = 0$
For a quadratic equation, $ax^2 + bx + c = 0$,
$D = b^2 - 4ac$
If $D = 0$, roots are real and equal
$2kx^2 - 40x + 25 = 0$
$\Rightarrow 1600 - 4 \times 2k \times 25 = 0$
$\Rightarrow k = 8$

(vi) $9x^2 - 24x + k = 0$
For a quadratic equation, $ax^2 + bx + c = 0$,
$D = b^2 - 4ac$
If $D = 0$, roots are real and equal
$9x^2 - 24x + k = 0$
$\Rightarrow D = 576 - 4 \times 9 \times k = 0$
$\Rightarrow k = \dfrac{576}{36} = 16$

(vii) $4x^2 - 3kx + 1 = 0$
For a quadratic equation, $ax^2 + bx + c = 0$,
$D = b^2 - 4ac$
If $D = 0$, roots are real and equal
$4x^2 - 3kx + 1 = 0$
$\Rightarrow D = 9k^2 - 4 \times 4 \times 1 = 0$
$\Rightarrow 9k^2 = 16$
$\Rightarrow k = \dfrac{4}{3}$

(viii) $x^2 - 2(5 + 2k)x + 3(7 + 10k) = 0$

For a quadratic equation, $ax^2 + bx + c = 0$,
$D = b^2 - 4ac$
If $D = 0$, roots are real and equal
$x^2 - 2(5 + 2k)x + 3(7 + 10k) = 0$
$\Rightarrow D = 4(5 + 2k)^2 - 4 \times 3(7 + 10k) = 0$
$\Rightarrow 100 + 16k^2 + 80k - 84 - 120k = 0$
$\Rightarrow 16k^2 - 40k + 16 = 0$
$\Rightarrow 2k^2 - 5k + 2 = 0$
$\Rightarrow 2k^2 - 4k - k + 2 = 0$
$\Rightarrow 2k(k - 2) - (k - 2) = 0$
$\Rightarrow (2k - 1)(k - 2) = 0$
$\Rightarrow k = 2, \frac{1}{2}$

(ix) $(3k + 1)x^2 + 2(k + 1)x + k = 0$
For a quadratic equation, $ax^2 + bx + c = 0$,
$D = b^2 - 4ac$
If $D = 0$, roots are real and equal
$(3k + 1)x^2 + 2(k + 1)x + k = 0$
$\Rightarrow D = 4(k + 1)^2 - 4k(3k + 1) = 0$
$\Rightarrow 4k^2 + 8k + 4 - 12k^2 - 4k = 0$
$\Rightarrow 2k^2 - k - 1 = 0$
$\Rightarrow 2k^2 - 2k + k - 1 = 0$
$\Rightarrow 2k(k - 1) + (k - 1) = 0$
$\Rightarrow (2k + 1)(k - 1) = 0$
$\Rightarrow k = 1, -\frac{1}{2}$

(x) $\mathrm{kx^2 + kx + 1 = -4x^2 - x}$
For a quadratic equation, $\mathrm{ax^2 + bx + c = 0}$,
$D = b^2 - 4ac$
If $D = 0$, roots are real and equal
$kx^2 + kx + 1 = -4x^2 - x$
$\Rightarrow (k + 4)x^2 + (k + 1)x + 1 = 0$
$\mathrm{D} = (k + 1)^2 - 4(k + 4) = 0$
$\Rightarrow k^2 + 2k + 1 - 4k - 16 = 0$
$\Rightarrow k^2 - 2k - 15 = 0$
$\Rightarrow k^2 - 5k + 3k - 15 = 0$
$\Rightarrow k(k - 5) + 3(k - 5) = 0$
$\Rightarrow (k + 3)(k - 5) = 0$
$\Rightarrow k = 5, -3$

(xi) $(k+1)x^2 + 2(k+3)x + (k+8) = 0$
For a quadratic equation, $ax^2 + bx + c = 0$,
$D = b^2 - 4ac$
If $D = 0$, roots are real and equal
$(k+1)x^2 + 2(k+3)x + (k+8) = 0$
$\Rightarrow D = 4(k+3)^2 - 4(k+1)(k+8) = 0$
$\Rightarrow 4k^2 + 36 + 24k - 4k^2 - 32 - 36k = 0$
$\Rightarrow 12k = 4$
$\Rightarrow k = \frac{1}{3}$

(xii) $x^2 - 2kx + 7x + \frac{1}{4} = 0$
For a quadratic equation, $ax^2 + bx + c = 0$,
$D = b^2 - 4ac$
If $D = 0$, roots are real and equal
$x^2 - 2kx + 7x + \frac{1}{4} = 0$
$\Rightarrow D = (7-2k)^2 - 4 \times \frac{1}{4} = 0$
$\Rightarrow 49 + 4k^2 - 28k - 1 = 0$
$\Rightarrow k^2 - 7k + 12 = 0$
$\Rightarrow k^2 - 4k - 3k + 12 = 0$
$\Rightarrow k(k-4) - 3(k-4) = 0$
$\Rightarrow (k-3)(k-4) = 0$
$\Rightarrow k = 3,4$

(xiii) $(k+1)x^2 - 2(3k+1)x + 8k + 1 = 0$
For a quadratic equation, $ax^2 + bx + c = 0$,
$D = b^2 - 4ac$
If $D = 0$, *roots are real and equal*
$(k+1)x^2 - 2(3k+1)x + 8k + 1 = 0$
$\Rightarrow D = 4(3k+1)^2 - 4(k+1)(8k+1) = 0$
$\Rightarrow 4 \times (9k^2 + 6k + 1) - 32k^2 - 4 - 36k = 0$
$\Rightarrow 36k^2 + 24k + 4 - 32k^2 - 4 - 36k = 0$
$\Rightarrow 4k(k-3) = 0$
$\Rightarrow k = 0,3$

(xiv) $5x^2 - 4x + 2 + k(4x^2 - 2x - 1) = 0$
For a quadratic equation, $ax^2 + bx + c = 0$,
$D = b^2 - 4ac$
If $D = 0$, roots are real and equal
$5x^2 - 4x + 2 + k(4x^2 - 2x - 1) = 0$

$\Rightarrow (5 + 4k)x^2 - (4 + 2k)x + 2 - k = 0$
$\Rightarrow D = (4 + 2k)^2 - 4 \times (5 + 4k)(2 - k) = 0$
$\Rightarrow 16 + 4k^2 + 16k + 16k^2 - 12k - 40 = 0$
$\Rightarrow 20k^2 - 4k - 24 = 0$
$\Rightarrow 5k^2 - k - 6 = 0$
$\Rightarrow 5k^2 - 6k + 5k - 6 = 0$
$\Rightarrow k(5k - 6) + (5k - 6) = 0$
$\Rightarrow (k + 1)(5k - 6) = 0$
$\Rightarrow k = -1, \frac{6}{5}$

(xv)$(4 - k)x^2 + (2\text{k} + 4)x + (8k + 1) = 0$
For a quadratic equation, $ax^2 + bx + c = 0$,
$D = b^2 - 4ac$
If $D = 0$, roots are real and equal
$(4 - k)x^2 + (2k + 4)x + (8k + 1) = 0$
$\Rightarrow D = (2k + 4)^2 - 4 \times (4 - k)(8k + 1) = 0$
$\Rightarrow 4k^2 + 16 + 16k + 32k^2 - 16 - 124k = 0$
$\Rightarrow 36k^2 - 108k = 0$
$\Rightarrow 36k(k - 3) = 0$
$\Rightarrow k = 0,3$

(xvi)$(2k + 1)x^2 + 2(k + 3)x + (k + 5) = 0$
For a quadratic equation, $ax^2 + bx + c = 0$,
$D = b^2 - 4ac$
If $D = 0$, roots are real and equal
$(2k + 1)x^2 + 2(k + 3)x + (k + 5) = 0$
$\Rightarrow D = 4(k + 3)^2 - 4 \times (2k + 1)(k + 5) = 0$
$\Rightarrow 4k^2 + 36 + 24k - 8k^2 - 20 - 44k = 0$
$\Rightarrow -4k^2 - 20k + 16 = 0$
$\Rightarrow k^2 + 5k - 4 = 0$
$\Rightarrow k = \frac{-5 \pm \sqrt{25 + 4 \times 4}}{2} = \frac{-5 \pm \sqrt{41}}{2}$

(xvii) $4x^2 - 2(k + 1)x + (k + 4) = 0$
For a quadratic equation, $ax^2 + bx + c = 0$,
$D = b^2 - 4ac$
If $D = 0$, roots are real and equal
$4x^2 - 2(k + 1)x + (k + 4) = 0$
$\Rightarrow D = 4(k + 1)^2 - 4 \times 4(k + 4) = 0$
$\Rightarrow 4k^2 + 8k + 4 - 16k - 64 = 0$
$\Rightarrow k^2 - 2k - 15 = 0$

$\Rightarrow k^2 - 5k + 3k - 15 = 0$
$\Rightarrow (k-5)(k+3) = 0$
$\Rightarrow k = -3, 5$

(xviii) $x^2 - 2(k+1)x + k^2 = 0$
For a quadratic equation, $ax^2 + bx + c = 0$,
$D = b^2 - 4ac$
If $D = 0$, roots are real and equal
$x^2 - 2(k+1)x + k^2 = 0$
$\Rightarrow D = 4(k+1)^2 - 4k^2 = 0$
$\Rightarrow 4k^2 + 8k + 4 - 4k^2 = 0$
$\Rightarrow k = -\frac{1}{2}$

(xix) $k^2x^2 - 2(k-1)x + 4 = 0$
For a quadratic equation, $ax^2 + bx + c = 0$,
$D = b^2 - 4ac$
If $D = 0$, roots are real and equal
$k^2x^2 - 2(k-1)x + 4 = 0$
$\Rightarrow D = 4(k-1)^2 - 4 \times 4k^2 = 0$
$\Rightarrow 4k^2 - 8k + 4 - 16k^2 = 0$
$\Rightarrow 12k^2 + 8k - 4 = 0$
$\Rightarrow 3k^2 + 2k - 1 = 0$
$\Rightarrow 3k^2 + 3k - k - 1 = 0$
$\Rightarrow 3k(k+1) - (k+1) = 0$
$\Rightarrow (3k-1)(k+1) = 0$
$\Rightarrow k = \frac{1}{3}, -1$

(xx) $(k+1)x^2 - 2(k-1)x + 1 = 0$
For a quadratic equation, $ax^2 + bx + c = 0$,
$D = b^2 - 4ac$
If $D = 0$, roots are real and equal
$(k+1)x^2 - 2(k-1)x + 1 = 0$
$\Rightarrow D = 4(k-1)^2 - 4 \times (k+1) = 0$
$\Rightarrow 4k^2 - 8k + 4 - 4k - 4 = 0$
$\Rightarrow 4k(k-3) = 0$
$\Rightarrow k = 0,3$

(xxi) $2x^2 + kx + 3 = 0$
For a quadratic equation, $ax^2 + bx + c = 0$,
$D = b^2 - 4ac$

If $D = 0$, roots are real and equal
$2x^2 + kx + 3 = 0$
$\Rightarrow D = k^2 - 4 \times 2 \times 3 = 0$
$\Rightarrow k^2 = 24$
$\Rightarrow k = 2\sqrt{6}$

(xxii) $kx(x-2) + 6 = 0$
For a quadratic equation, $ax^2 + bx + c = 0$,
$D = b^2 - 4ac$
If $D = 0$, roots are real and equal
$kx(x-2) + 6 = 0$
$\Rightarrow kx^2 - 2kx + 6 = 0$
$\Rightarrow D = 4k^2 - 4 \times 6 \times k = 0$
$\Rightarrow 4k(k-6) = 0$
$\Rightarrow k = 0,6$ but k can't be 0 as it is the coefficient of x^2, thus $k = 6$

(xxiii) $x^2 - 4kx + k = 0$
For a quadratic equation, $ax^2 + bx + c = 0$,
$D = b^2 - 4ac$
If $D = 0$, roots are real and equal
$x^2 - 4kx + k = 0$
$\Rightarrow D = 16k^2 - 4k = 0$
$\Rightarrow 4k(4k-1) = 0$
$\Rightarrow k = 0, \frac{1}{4}$

(xxiv) $kx(x - 2\sqrt{5}) + 10 = 0$
For a quadratic equation, $ax^2 + bx + c = 0$,
$D = b^2 - 4ac$
If $D = 0$, roots are real and equal
$kx(x - 2\sqrt{5}) + 10 = 0$
$\Rightarrow kx^2 - 2\sqrt{5}kx + 10 = 0$
$\Rightarrow D = 4 \times 5k^2 - 4 \times k \times 10 = 0$
$\Rightarrow k^2 = 2k$
$\Rightarrow k = 2$

(xv) $px(x-3) + 9 = 0$
For a quadratic equation, $ax^2 + bx + c = 0$,
$D = b^2 - 4ac$
If $D = 0$, roots are real and equal
$px(x-3) + 9 = 0$
$\Rightarrow px^2 - 3px + 9 = 0$

$\Rightarrow D = 9p^2 - 4 \times 9 \times p = 0$
$\Rightarrow p = 4$

(xxvi) $4x^2 + px + 3 = 0$
For a quadratic equation, $ax^2 + bx + c = 0$,
$D = b^2 - 4ac$
If $D = 0$, roots are real and equal
$4x^2 + px + 3 = 0$
$\Rightarrow D = p^2 - 4 \times 4 \times 3 = 0$
$\Rightarrow p^2 = 48$
$\Rightarrow p = 4\sqrt{3}$

Q3. In the following, determine the set of values of k for which the given quadratic equation has real roots:
(i) $2x^2 + 3x + k = 0$
(ii) $2x^2 + kx + 3 = 0$
(iii) $2x^2 - 5x - k = 0$
(iv) $kx^2 + 6x + 1 = 0$
(v) $x^2 - kx + 9 = 0$
(vi) $2x^2 + kx + 2 = 0$
(vii) $3x^2 + 2x + k = 0$
(viii) $4x^2 - 3kx + 1 = 0$
(ix) $2x^2 + kx - 4 = 0$

Solution:

(i) $2x^2 + 3x + k = 0$
For a quadratic equation, $ax^2 + bx + c = 0$,
$D = b^2 - 4ac$
If $D \geq 0$, roots are real
$2x^2 + 3x + k = 0$
$\Rightarrow D = 9 - 4 \times 2 \times k$
$\Rightarrow 9 - 8k \geq 0$
$\Rightarrow k \leq \frac{9}{8}$

(ii) $2x^2 + kx + 3 = 0$
For a quadratic equation, $ax^2 + bx + c = 0$,
$D = b^2 - 4ac$
If $D \geq 0$, roots are real
$2x^2 + kx + 3 = 0$
$\Rightarrow D = k^2 - 4 \times 2 \times 3$
$D \geq 0$
$\Rightarrow k^2 - 24 \geq 0$

$\Rightarrow (k+2\sqrt{6})(k-2\sqrt{6}) \geq 0$
$Thus, k \leq -2\sqrt{6}\ or\, k \geq 2\sqrt{6}$

(iii) $2x^2 - 5x - k = 0$
For a quadratic equation, $ax^2 + bx + c = 0$,
$D = b^2 - 4ac$
$If\, D \geq 0, roots\ are\ real$
$2x^2 - 5x - k = 0$
$\Rightarrow D = 25 - 8k$
$D \geq 0$
$\Rightarrow 25 - 8k \geq 0$
$\Rightarrow k \leq \frac{25}{8}$

(iv) $kx^2 + 6x + 1 = 0$
For a quadratic equation, $ax^2 + bx + c = 0$,
$D = b^2 - 4ac$
If $D \geq 0$, roots are real
$kx^2 + 6x + 1 = 0$
$\Rightarrow D = 36 - 4k$
$\Rightarrow 36 - 4k \geq 0$
$\Rightarrow k \leq 9$

(v) $x^2 - kx + 9 = 0$
For a quadratic equation, $ax^2 + bx + c = 0$,
$D = b^2 - 4ac$
If $D \geq 0$, roots are real
$x^2 - kx + 9 = 0$
$\Rightarrow D = k^2 - 36$
$\Rightarrow k^2 - 36 \geq 0$
$\Rightarrow (k-6)(k+6) \geq 0$
$\Rightarrow k \geq 6\ or\, k \leq -6$

(vi) $2x^2 + kx + 2 = 0$
For a quadratic equation, $ax^2 + bx + c = 0$,
$D = b^2 - 4ac$
If $D \geq 0$, roots are real
$2x^2 + kx + 2 = 0$
$\Rightarrow D = k^2 - 4 \times 4$
$\Rightarrow k^2 - 16 \geq 0$
$\Rightarrow (k+4)(k-4) \geq 0$
$\Rightarrow k \geq 4\ or\, k \leq -4$

(vii) $3x^2 + 2x + k = 0$
For a quadratic equation, $ax^2 + bx + c = 0$,
$D = b^2 - 4ac$
If $D \geq 0$, roots are real
$3x^2 + 2x + k = 0$
$\Rightarrow D = 4 - 12k$
$\Rightarrow 4 - 12k \geq 0$
$\Rightarrow k \leq \frac{1}{3}$

(viii) $4x^2 - 3kx + 1 = 0$
For a quadratic equation, $ax^2 + bx + c = 0$,
$D = b^2 - 4ac$
If $D \geq 0$, roots are real
$4x^2 - 3kx + 1 = 0$
$\Rightarrow D = 9k^2 - 16$
$\Rightarrow 9\text{k}^2 - 16 \geq 0$
$\Rightarrow (3k - 4)(3k + 4) \geq 0$
$\Rightarrow k \leq -\frac{4}{3}\, or\, k \geq \left(\frac{4}{3}\right)$

(ix) $2x^2 + kx - 4 = 0$
For a quadratic equation, $ax^2 + bx + c = 0$,
$D = b^2 - 4ac$
If $D \geq 0$, roots are real
$2x^2 + kx - 4 = 0$
$\Rightarrow D = k^2 + 4 \times 2 \times 4 = k^2 + 32$
Thus, D is always greater than 0 for all values of k.

Q4. For what value of k, $(4 - k)x^2 + (2k + 4)x + (8k + 1) = 0$, is a perfect square.

Solution:

$(4 - k)x^2 + (2k + 4)x + (8k + 1) = 0$
For the above expression to be a perfect square, $D = b^2 - 4ac = 0$
$\Rightarrow (2k + 4)^2 - 4 \times (4 - k)(8k + 1) = 0$
$\Rightarrow 4k^2 + 16k + 16 + 32k^2 - 124k - 16 = 0$
$\Rightarrow 36k^2 - 108k = 0$
$\Rightarrow 36k(k - 3) = 0$
$\Rightarrow k = 0{,}3$

Q5. Find the least positive value of k for which the equation $x^2 + kx + 4 = 0$ has real roots.

Solution:

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If $D \geq 0$, roots are real

$x^2 + kx + 4 = 0$

$\Rightarrow D = k^2 - 16$

Thus, $k^2 - 16 \geq 0$

$\Rightarrow k \geq 4$ or $k \leq -4$

Thus, least positive value of k is 4.

Q6. Find the values of k for which the given quadratic equation has real and distinct roots:

(i) $kx^2 + 2x + 1 = 0$

(ii) $kx^2 + 6x + 1 = 0$

(iii) $x^2 - kx + 9 = 0$

Solution:

(i) For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If $D > 0$, roots are real and distinct

$kx^2 + 2x + 1 = 0$

$\Rightarrow D = 4 - 4k$

$\Rightarrow 4 - 4k > 0$

$\Rightarrow k < 1$

(ii) $kx^2 + 6x + 1 = 0$

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If $D > 0$, roots are real and distinct

$kx^2 + 6x + 1 = 0$

$\Rightarrow D = 36 - 4k$

$\Rightarrow 36 - 4k > 0$

$\Rightarrow k < 9$

(iii) $x^2 - kx + 9 = 0$

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If $D > 0$, roots are real and distinct $x^2 - kx + 9 = 0$

$\Rightarrow D = k^2 - 36$

$\Rightarrow k^2 - 36 > 0$

$\Rightarrow (k + 6)(k - 6) > 0$

$\Rightarrow k < -6\ or\ k > 6$

Q7. If the roots of the equation $(b - c)x^2 + (c - a)x + (a - b) = 0$ are equal, then prove that $2b = a + c$.

Solution:

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If $D = 0$, roots are equal

$(b-c)x^2 + (c-a)x + (a-b) = 0$

$\Rightarrow (c-a)^2 - 4(b-c)(a-b) = 0$

$\Rightarrow c^2 + a^2 - 2ac + 4b^2 - 4ab - 4cb + 4ac = 0$

$\Rightarrow a^2 + 4b^2 + c^2 + 2ac - 4ab - 4bc = 0$

$\Rightarrow (a - 2b + c)^2 = 0$

$\Rightarrow 2\,b = a + c$

Q8. If the roots of the equation $(a^2 + b^2)x^2 - 2(ab + cd)x + (c^2 + d^2) = 0$ are equal, prove that $\frac{a}{b} = \frac{c}{d}$.

Solution:

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If $D = 0$, roots are equal. For the given equation D would be -

$(a^2 + b^2) - 2(ab + cd) + (c^2 + d^2) = 0$

$\Rightarrow 4(ac + bd)^2 - 4(a^2 + b^2)(c^2 + d^2) = 0$

$\Rightarrow a^2c^2 + b^2d^2 + 2acbd - a^2d^2 - a^2c^2 - b^2d^2 - b^2c^2 = 0$

$\Rightarrow a^2d^2 + b^2c^2 - 2abcd = 0$

$\Rightarrow (ad - bc) = 0$

$\Rightarrow \frac{a}{b} = \frac{c}{d}$

Q9. If the roots of the equations $ax^2 + 2bx + c = 0$ and $bx^2 - 2\sqrt{ac}x + b = 0$ are simultaneously real, then prove that $b^2 = ac$.

Solution:

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If $D \geq 0$, roots are real

$ax^2 + 2bx + c = 0$

$\Rightarrow 4\,b^2 - 4ac \geq 0$

$\Rightarrow b^2 \geq ac$.......*(1)*

$bx^2 - 2\sqrt{ac}x + b = 0$

$\Rightarrow 4ac - 4\,b^2 \geq 0$

$\Rightarrow b^2 \leq ac$...........*(2)*

For both (1) and (2) to be true

$\Rightarrow b^2 = ac$

Q10. If p, q are real and $p \neq q$, then show that the roots of the equation $(p-q)x^2 + 5(p+q)x - 2(p-q) = 0$ are real and unequal

Solution:

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If $D > 0$, roots are real and unequal.

$(p - q)x^2 + 5(p + q)x - 2(p - q) = 0$

$\Rightarrow D = 25(p + q)^2 + 8(p - q)^2$

Thus $D > 0$ for all p and q as sum of two squares is always positive.

Q11. If the roots of the equation $(c^2 - ab)x^2 - 2(a^2 - bc)x + b^2 - ac = 0$ are equal, prove that either $a = 0$ or $a^3 + b^3 + c^3 = 3abc$.

Solution:

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If $D = 0$, roots are equal

Given, roots of $(c^2 - ab)x^2 - 2(a^2 - bc)x + b^2 - ac = 0$ are equal.

$\therefore \mathrm{D} = 0$

$\Rightarrow [2(a^2 - bc)]^2 - 4(c^2 - ab)(b^2 - ac) = 0$

$\Rightarrow 4(a^2 - bc)^2 - 4(c^2 - ab)(b^2 - ac) = 0$

$\Rightarrow a^4 + b^2c^2 - 2a^2bc - b^2c^2 - a^2bc + ab^3 + ac^3 = 0$

$\Rightarrow a(a^3 + b^3 + c^3 - 3abc) = 0$

$\Rightarrow a = 0$ or $a^3 + b^3 + c^3 = 3abc$.

Q12. Show that the equation $2(a^2 + b^2)x^2 + 2(a + b)x + 1 = 0$ has no real roots, when $a \neq b$.

Solution:

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If $D < 0$, roots are not real.

$2(a^2 + b^2)x^2 + 2(a + b)x + 1 = 0$

$\Rightarrow D = 4(a + b)^2 - 8(a^2 + b^2)$

$\Rightarrow D = 4a^2 + 4b^2 + 8ab - 8a^2 - 8b^2$

$\Rightarrow D = -4(a^2 + b^2 - 2ab) = -4(a - b)^2$

Thus, $\mathrm{D} < 0$ for all values of a and b.

$\therefore$ Roots are not real.

Q13. Prove that both the roots of the equation $(x - a)(x - b) + (x - b)(x - c) + (x - c)(x - a) = 0$ are real but they are equal only when $a = b = c$.

Solution:

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If $D > 0$, roots are real.

$(x - a)(x - b) + (x - b)(x - c) + (x - c)(x - a) = 0$

$\Rightarrow x^2-(a+b)x+ab+x^2-(b+c)x+bc+x^2-(a+c)x+ac=0$

$\Rightarrow 3x^2-2(a+b+c)x+ab+bc+ac=0$

$\Rightarrow D=4(a+b+c)^2-12(ab+bc+ac)$

$\Rightarrow D=a^2+b^2+c^2+2ab+2ac+2bc-3ab-3bc-3ac$

$\Rightarrow D=\frac{1}{2}\times(2a^2+2b^2+2c^2-2ab-2ac-2bc)$

$\Rightarrow D=\frac{1}{2}\times((a-b)^2+(b-c)^2+(c-a)^2)$

Thus, D is always greater than 0 , and the roots are real

Now, when $a=b=c$,

$D=0$, thus the roots are equal when $a=b=c$.

Q14. If a, b, c are real numbers such that $ac \neq 0$, then show that at least one of the equations $ax^2+bx+c=0$ and $-ax^2+bx+c=0$ has real roots.

Solution:

For a quadratic equation, $ax^2+bx+c=0$,

$D=b^2-4ac$

If $D>0$, roots are real.

$ax^2+bx+c=0$

$\Rightarrow D=b^2-4ac$

If $D>0$ then $b^2>4ac$

$-ax^2+bx+c=0$

$\Rightarrow D=b^2+4ac$

If $D>0, b^2>-4ac$

Thus, one of the equation has real roots.

Q15. If the equation $(1+m^2)x^2+2mcx+(c^2-a^2)=0$ has equal roots, prove that $c^2=a^2(1+m^2)$.

Solution:

For a quadratic equation, $ax^2+bx+c=0$,

$D=b^2-4ac$

If $D=0$, roots are equal

$(1+m^2)x^2+2mcx+(c^2-a^2)=0$

$\Rightarrow D=4m^2c^2-4(c^2-a^2)(1+m^2)=0$

$\Rightarrow m^2c^2-c^2+a^2-c^2m^2+a^2m^2=0$

$\Rightarrow c^2=a^2(1+m^2)$

Q16. Find the values of k for which the quadratic equation $(3k+1)x^2+2(k+1)x+1=0$ has equal roots. Also, find these roots.

Solution:

For a quadratic equation, $ax^2+bx+c=0$,

$D=b^2-4ac$

If $D = 0$, roots are equal
$(3k + 1)x^2 + 2(k + 1)x + 1 = 0$
$\Rightarrow D = 4(k + 1)^2 - 4(3k + 1) = 0$
$\Rightarrow k^2 + 2k + 1 - 3k - 1 = 0$
$\Rightarrow k(k - 1) = 0$
$\Rightarrow k = 0,1$
When $k = 0$,
Eq. $-x^2 + 2x + 1 = 0$
$\Rightarrow (x + 1)^2 = 0$
$\Rightarrow x = -1$
When $\mathrm{k} = 1$,
Eq. $-4x^2 + 4x + 1 = 0$
$\Rightarrow (2x + 1)^2 = 0$
$\Rightarrow x = -\frac{1}{2}$

Q17. Find the values of p for which the quadratic equation $(2p + 1)x^2 - (7p + 2)x + (7p - 3) = 0$ has equal roots. Also, find these roots.

Solution:
For a quadratic equation, $ax^2 + bx + c = 0$,
$D = b^2 - 4ac$
If $D = 0$, roots are equal
$(2p + 1)x^2 - (7p + 2)x + (7p - 3) = 0$
$\Rightarrow D = (7p + 2)^2 - 4(7p - 3)(2p + 1) = 0$
$\Rightarrow 49p^2 + 28p + 4 - 56p^2 + 12 - 4p = 0$
$\Rightarrow 7p^2 - 24p - 16 = 0$
$\Rightarrow 7p^2 - 28p + 4p - 16 = 0$
$\Rightarrow 7p(p - 4) + 4(p - 4) = 0$
$\Rightarrow (7p + 4)(p - 4) = 0$
$\Rightarrow p = -\frac{4}{7}, 4$

Q18. If -5 is a root of the quadratic equation $2x^2 + px - 15 = 0$ and the quadratic equation $p(x^2 + x) + k = 0$ has equal roots, find the value of k.

Solution:
For a quadratic equation, $ax^2 + bx + c = 0$,
$D = b^2 - 4ac$
If $D = 0$, roots are equal
Given, -5 is a root of the quadratic equation $2x^2 + px - 15 = 0$
$\Rightarrow 2 \times 25 - 5p - 15 = 0$
$\Rightarrow 35 = 5p$
$\Rightarrow p = 7$

Now, the quadratic equation $p(x^2 + x) + k = 0$ has equal roots

$\Rightarrow 7x^2 + 7x + k = 0$ has equal roots

$\Rightarrow \mathrm{D} = 49 - 28k = 0$

$\Rightarrow k = \frac{49}{28} = \frac{7}{4}$

Q19. If 2 is a root of the quadratic equation $3x^2 + px - 8 = 0$ and the quadratic equation $4x^2 - 2px + k = 0$ has equal roots, find the value of k.

Solution:

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If $D = 0$, roots are equal

Given, 2 is a root of the quadratic equation $3x^2 + px - 8 = 0$

$\Rightarrow 3 \times 4 + 2p - 8 = 0$

$\Rightarrow 2p = -4$

$\Rightarrow p = -2$

Now, the quadratic equation $4x^2 - 2px + k = 0$ has equal roots

$\Rightarrow 4x^2 + 4x + k = 0$ has equal roots

$\Rightarrow D = 16 - 16k = 0$

$\Rightarrow k = 1$

Q20. If 1 is a root of the quadratic equation $3x^2 + ax - 2 = 0$ and the quadratic equation $a(x^2 + 6x) - b = 0$ has equal roots, find the value of b.

Solution:

For a quadratic equation, $ax^2 + bx + c = 0$,

$D = b^2 - 4ac$

If $D = 0$, roots are equal

Given, 1 is a root of the quadratic equation $3x^2 + ax - 2 = 0$

$\Rightarrow 3 + a - 2 = 0$

$\Rightarrow \mathrm{a} = -1$

Now, the quadratic equation $a(x^2 + 6x) - b = 0$ has equal roots

$\Rightarrow x^2 + 6x + b = 0$ has equal roots

$\Rightarrow D = 36 - 4b = 0$

$\Rightarrow b = 9$

Q21. Find the value of p for which the quadratic equation: $(p + 1)x^2 - 6(p + 1)x + 3(p + 9) = 0$, where, $p \neq -1$ has equal roots. Hence, find the roots of the equation.

Solution:

Note: For a quadratic equation, $ax^2 + bx + c = 0$, we have $D = b^2 - 4ac$.

If $D = 0$, then the roots of the quadratic equation are equal.

Therefore, $(p + 1)x^2 - 6(p + 1)x + 3(p + 9) = 0$ will have equal roots when,

$\Rightarrow D = 0$

$\Rightarrow b^2 - 4ac = 0$
$\Rightarrow b^2 = 4ac$
Here, $b = -6(p+1)$,
$a = (p+1)$
and, $c = 3(p+9) \Rightarrow \{-6(p+1)\}^2 = 4 \times (p+1) \times 3(p+9)$
$\Rightarrow 36(p+1)(p+1) = 12(p+1)(p+9) \Rightarrow 3(p+1) = (p+9)$
$\Rightarrow 3p + 3 - p - 9 = 0$
$\Rightarrow 2p - 6 = 0$
$\Rightarrow p = \frac{6}{2}$
$\Rightarrow p = 3$
Thus, the value of p is 3
Now, putting the value of p in $(p+1)x^2 - 6(p+1)x + 3(p+9) = 0$, we get,
$\Rightarrow 4x^2 - 24x + 36 = 0$
On taking 4 common, we get,
$\Rightarrow x^2 - 6x + 9 = 0$
$\Rightarrow (x-3)^2 = 0$
$\Rightarrow x = 3$
Thus, the root of the given equation is $x = 3$

Exercise 8.7

Q1. Find two consecutive numbers whose squares have the sum of 85

Solution:

Let the consecutive numbers be ' a ' and $a+1$.
Given, sum of squares is 85
$\Rightarrow a^2 + (a+1)^2 = 85$
$\Rightarrow a^2 + a^2 + 2a + 1 = 85$
$\Rightarrow a^2 + a - 42 = 0$
$\Rightarrow a^2 + 7a - 6a - 42 = 0$
$\Rightarrow a(a+7) - 6(a+7) = 0$
$\Rightarrow (a-6)(a+7) = 0$
$\Rightarrow a = 6, -7$
Numbers are, 6, 7 or $-7, -6$.

Q2. Divide 29 into two parts so that the sum of the squares of the parts is 425.

Solution:

Let one of the numbers be ' a '.
Given, sum of two numbers is 29 and the sum of their squares is 425
$\Rightarrow a^2 + (29-a)^2 = 425$
$\Rightarrow a^2 + 841 + a^2 - 58a = 425$
$\Rightarrow a^2 - 29a + 416 = 0$

$\Rightarrow a^2 - 16a - 13a + 208 = 0$
$\Rightarrow a(a - 16) - 13(a - 16) = 0$
$\Rightarrow (a - 13)(a - 16) = 0$
$\Rightarrow a = 13,16$

Q3. Two squares have sides x cm and $(x + 4)$cm. The sum of their areas is 656 cm^2. Find the sides of the squares.

Solution:

Area of a square = side × side
Given, squares have sides x cm and $(x + 4)$cm. The sum of their areas is 656 cm^2
$\Rightarrow x^2 + (x + 4)^2 = 656$
$\Rightarrow \mathrm{x}^2 + \mathrm{x}^2 + 16 + 8\mathrm{x} = 656$
$\Rightarrow x^2 + 4x - 320 = 0$
$\Rightarrow x^2 - 16x + 20x - 320 = 0$
$\Rightarrow x(x - 16) + 20(x - 16) = 0$
$\Rightarrow (x - 16)(x + 20) = 0$
$\Rightarrow x = 16, -20$
The sides are 16,20.

Q4. The sum of two numbers is 48 and their product is 432. Find the numbers.

Solution:

Let the numbers be ' a ' and ' b '.
Given, sum of two numbers is 48 and their product is 432 .
$\Rightarrow a + b = 48$
$\Rightarrow a = 48 - b$
Also, $ab = 432$
$\Rightarrow 48\, b - b^2 = 432$
$\Rightarrow b^2 - 48b + 432 = 0$
$\Rightarrow b^2 - 36b - 12b + 432 = 0$
$\Rightarrow b(b - 36) - 12(\, b - 36) = 0$
$\Rightarrow (b - 12)(b - 36) = 0$
$\Rightarrow b = 12,36$

Q5. If an integer is added to its square, the sum is 90. Find the integer with the help of quadratic equation.

Solution:

Let the integer be ' a '.
Given, an integer is added to its square, the sum is 90
$\Rightarrow a + a^2 = 90$
$\Rightarrow a^2 + 10a - 9a - 90 = 0$
$\Rightarrow a(a + 10) - 9(a + 10) = 0$
$\Rightarrow (a - 9)(a + 10) = 0$

$\Rightarrow a = -10,9$

Q6. Find the whole number which when decreased by 20 is equal to 69 times the reciprocal of the number.

Solution:

Let the number be ' a '

$\Rightarrow \text{a} - 20 = 69/\text{a}$

$\Rightarrow a^2 - 20a - 69 = 0$

$\Rightarrow a^2 - 23a + 3a - 69 = 0$

$\Rightarrow a(a - 23) + 3(a - 23) = 0$

$\Rightarrow (a + 3)(a - 23) = 0$

$\Rightarrow a = 23 \; or -3$

Whole number is 23.

Q7. Find two consecutive natural numbers whose product is 20.

Solution:

Let the consecutive numbers be $a, a + 1$.

$\Rightarrow a(a + 1) = 20$

$\Rightarrow a^2 + a - 20 = 0$

$\Rightarrow a^2 + 5a - 4a - 20 = 0$

$\Rightarrow a(a + 5) - 4(a + 5) = 0$

$\Rightarrow (a - 4)(a + 5) = 0$

$\Rightarrow a = 4$ is a natural number

Thus the numbers are 4 and 5.

Q8. The sum of the squares of two consecutive odd positive integers is 394. Find then.

Solution:

Let the consecutive odd integers be a, a +2

$\Rightarrow a^2 + (a + 2)^2 = 394$

$\Rightarrow a^2 + a^2 + 4a + 4 = 394$

$\Rightarrow a^2 + 2a - 195 = 0$

$\Rightarrow a^2 + 15a - 13a - 195 = 0$

$\Rightarrow a(a + 15) - 13(a + 15) = 0$

$\Rightarrow (a - 13)(a + 15) = 0$

$\Rightarrow a = 13, -15$

The numbers are 13 and 15.

Q9. The sum of two numbers is 8 and 15 times the sum of their reciprocals is also 8. Find the numbers.

Solution:

Let the numbers be ' a ' and ' b '.

Given, sum of two numbers is 8 and 15 times the sum of their reciprocals is also 8 .

$\Rightarrow a + b = 8$

$\Rightarrow a = 8 - b$

Also, $15 \times \left(\frac{1}{a} + \frac{1}{b}\right) = 8$

$$\Rightarrow \frac{1}{a} + \frac{1}{b} = \frac{8}{15}$$

$$\Rightarrow \frac{1}{a} + \frac{1}{8-a} = \frac{8}{15}$$

$\Rightarrow 15(8 - a + a) = 8(8a - a^2)$

$\Rightarrow a^2 - 8a + 15 = 0$

$\Rightarrow a^2 - 5a - 3a + 15 = 0$

$\Rightarrow a(a - 5) - 3(a - 5) = 0$

$\Rightarrow (a - 3)(a - 5) = 0$

$\Rightarrow a = 3,5.$

Q10. The sum of a number and its positive square root is 6/25. Find the number.

Solution:

Given: The sum of a number and its positive square root is 6/25. To find: the number.Solution:Let the number be ' a '.

$$\Rightarrow a + \sqrt{a} = \frac{6}{25}$$

$$\Rightarrow \sqrt{a} = \left(\frac{6}{25}\right) - a$$

Squaring both sides

$$\Rightarrow a = \frac{36}{625} + a^2 - \frac{12a}{25}$$

$\Rightarrow a^2 - \frac{37a}{25} + \frac{36}{625} =$ 0factorise by splitting the middle term.

$$\Rightarrow a^2 - \frac{a}{25} - \frac{36a}{25} + \frac{36}{625} = 0$$

$$\Rightarrow a\left(a - \frac{1}{25}\right) - \left(\frac{36}{25}\right) \times \left(a - \frac{1}{25}\right) = 0$$

$$\Rightarrow \left(a - \frac{36}{25}\right)\left(a - \frac{1}{25}\right) = 0$$

$$\Rightarrow a = \frac{36}{25}, \frac{1}{25}$$

But only $\frac{1}{25}$ is possible as its sum with its positive root is $\frac{6}{25}$. Hence the number is $\frac{1}{25}$.

Q11. The sum of a number and its square is $\frac{63}{4}$, find the numbers.

Solution:

Let the number be 'a'

$$\Rightarrow a + a^2 = \frac{63}{4}$$

$\Rightarrow 4a^2 + 4a - 63 = 0$

$\Rightarrow 4a^2 + 18a - 14a - 63 = 0$
$\Rightarrow 2a(2a + 9) - 7(2a + 9) = 0$
$\Rightarrow (2a - 7)(2a + 9) = 0$
$\Rightarrow a = \frac{7}{2}$ or $-\frac{9}{2}$.

Q12. There are three consecutive integers such that the square of the first increased by the product of the other two gives 154. What are the integers?

Solution:

Let the three consecutive numbers be a, a +1 , $a + 2$
Given, there are three consecutive integers such that the square of the first increased by the product of the other two gives 154.
$\Rightarrow a^2 + (a + 1)(a + 2) = 154$
$\Rightarrow 2a^2 + 3a + 2 = 154$
$\Rightarrow 2a^2 + 3a - 152 = 0$
$\Rightarrow 2a^2 + 19a - 16a - 152 = 0$
$\Rightarrow a(2a + 19) - 8(2a + 19) = 0$
$\Rightarrow (a - 8)(2a + 19) = 0$
$Thus, a = 8$
Numbers are 8, 9, 10.

Q13. The product of two successive integral multiples of 5 is 300. Determine the multiples.

Solution:

Let the successive integral multiples of 5 be $a, a + 5$.
$\Rightarrow a(a + 5) = 300$
$\Rightarrow a^2 + 5a - 300 = 0$
$\Rightarrow a^2 + 20a - 15a - 300 = 0$
$\Rightarrow a(a + 20) - 15(a + 20) = 0$
$\Rightarrow (a + 20)(a - 15) = 0$
$\Rightarrow a = -20, 15$
Numbers are, $-20, -15$ or $15, 20$

Q14. The sum of the squares of two numbers is 233 and one of the numbers is 3 less than twice the other number. Find the numbers.

Solution:

Given: The sum of the squares of two numbers is 233 and one of the numbers is 3 less than twice the other number. To find: the numbers. Let one of the numbers be a.
Given, sum of the squares of two numbers is 233 and one of the numbers is 3 less than twice the other number.
2^{nd} number = 2a − 3
According to given condition,
$a^2 + (2a - 3)^2 = 233$ Apply the formula $(x - y)^2 = x^2 + y^2 - 2xy$ on $(2a - 3)^2$
$\Rightarrow a^2 + 4a^2 + 9 - 12a = 233 \Rightarrow a^2 + 4a^2 + 9 - 12a - 233 = 0$

$\Rightarrow 5a^2 - 12a - 224 = 0$
$\Rightarrow 5a^2 - 40a + 28a - 224 = 0$
$\Rightarrow 5a(a-8) + 28(a-8) = 0$
$\Rightarrow (5a+28)(a-8) = 0 \Rightarrow (5a+28) = 0$ and $(a-8) = 0$
$\Rightarrow a = -\frac{28}{5}$ and $a = 8$ To satisfy the given conditions a will be 8.2^{nd} number $= 2(8) - 3 = 16 - 3 = 13$
Thus the numbers are 8,13 .

Q15. Find the consecutive even integers whose squares have the sum 340.

Solution:

Let the consecutive even integers be ' a ' and a + 2
$\Rightarrow a^2 + (a+2)^2 = 340$
$\Rightarrow 2a^2 + 4a - 336 = 0$
$\Rightarrow a^2 + 2a - 168 = 0$
$\Rightarrow a^2 + 14a - 12a - 168 = 0$
$\Rightarrow a(a+14) - 12(a+14) = 0$
$\Rightarrow (a-12)(a+14) = 0$
Thus, $a = 12$ or -14
Consecutive even integers are 12,14 or -14 , -12.

Q16. The difference of two numbers is 4 . If the difference of their reciprocals is $\frac{4}{21}$, find the numbers.

Solution:

Let the numbers be 'a' and ' b '.
Given, difference of two numbers is 4 and difference of their reciprocals is $\frac{4}{21}$
$\Rightarrow a - b = 4$
$\Rightarrow a = b + 4$
$and \frac{1}{b} - \frac{1}{a} = \frac{4}{21}$
$$\Rightarrow \frac{1}{b+4} - \frac{1}{b} = -\frac{4}{21}$$
$\Rightarrow 21(b - b - 4) = -4(b^2 + 4b)$
$\Rightarrow b^2 + 4b - 21 = 0$
$\Rightarrow b^2 + 7b - 3b - 21 = 0$
$\Rightarrow b(b+7) - 3(b+7) = 0$
$\Rightarrow (b-3)(b+7) = 0$
$\Rightarrow b = 3, -7$
Numbers are , 3, 7 or $-7, -3$.

Q17. Find two natural numbers which differ by 3 and whose squares have the sum 117.

Solution:

Let one of the natural numbers be 'a'
Given, the numbers differ by 3.
$\Rightarrow 2^{\text{nd}}$ number $= a + 3$
$\Rightarrow a^2 + (a+3)^2 = 117$
$\Rightarrow a^2 + a^2 + 6a + 9 = 117$
$\Rightarrow a^2 + 3a - 54 = 0$
$\Rightarrow a^2 + 9a - 6a - 54 = 0$
$\Rightarrow a(a+9) - 6(a+9) = 0$
$\Rightarrow (a-6)(a+9) = 0$
$\Rightarrow a = 6, -9$
Thus, the numbers are 6,9.

Q18. The sum of the squares of three consecutive natural numbers is 149. Find the numbers.

Solution:

Let the three consecutive natural numbers be 'a', 'a+1' and 'a +2 '
$\Rightarrow a^2 + (a+1)^2 + (a+2)^2 = 149$
$\Rightarrow a^2 + a^2 + 2a + 1 + a^2 + 4a + 4 = 149$
$\Rightarrow 3a^2 + 6a - 144 = 0$
$\Rightarrow a^2 + 2a - 48 = 0$
$\Rightarrow a^2 + 8a - 6a - 48 = 0$
$\Rightarrow a(a+8) - 6(a+8) = 0$
$\Rightarrow (a-6)(a+8) = 0$
$\Rightarrow a = 6$ or $a = -8$, however $a = -8$ is not possible as -8 is not a natural number.

Q19. The sum of two numbers is 16. The sum of their reciprocals is $\frac{1}{3}$. Find the numbers.

Solution:

Let the numbers be ' a ' and ' b '
Given, sum of two numbers is 16. The sum of their reciprocals is $\frac{1}{3}$.
$\Rightarrow a + b = 16$
$\Rightarrow a = 16 - b$
Also, $\frac{1}{a} + \frac{1}{b} = \frac{1}{3}$

$$\Rightarrow \frac{1}{16-b} + \frac{1}{b} = \frac{1}{3}$$

$\Rightarrow 3(b + 16 - b) = 16b - b^2$
$\Rightarrow b^2 - 16b + 48 = 0$
$\Rightarrow b^2 - 12b - 4b + 48 = 0$
$\Rightarrow b(b-12) - 4(b-12) = 0$
$\Rightarrow (b-4)(b-12) = 0$
$\Rightarrow b = 4, 12$
Numbers are 4, 12.

Q20. Determine two consecutive multiples of 3 whose product is 270.

Solution:

Let the consecutive multiples of 3 be $a, a+3$

$\Rightarrow a(a+3)=270$

$\Rightarrow a^2+3a-270=0$

$\Rightarrow a^2+18a-15a-270=0$

$\Rightarrow a(a+18)-15(a+18)=0$

$\Rightarrow (a-15)(a+18)=0$

$\Rightarrow a=15$

Numbers are 15, 18.

Q21. The sum of a number and its reciprocal is $\frac{17}{4}$. Find the number.

Solution:

Let the number be a.

$$a+\frac{1}{a}=\frac{17}{4}$$

$$\Rightarrow 4\left(a+\frac{1}{a}\right)=1$$

$$\Rightarrow 4\left(\frac{a^2+1}{a}\right)=17$$

$\Rightarrow 4a^2+4-17a=0$

$\Rightarrow 4a^2-16a-a+4=0$

$\Rightarrow 4\text{a}(\text{a}-4)-(\text{a}-4)=0$

$\Rightarrow (4a-1)(a-4)=0$

$\Rightarrow a=\frac{1}{4}$ or 4

Q22. A two-digit number is such that the product of its digits is 8. When 18 is subtracted from the number, the digits interchange their places. Find number.

Solution:

Let the ones digit be ' a ' and tens digit be ' b '.

Given, two-digit number is such that the product of its digits is 8 .

$\Rightarrow ab=8$--- (1)

Also, when 18 is subtracted from the number, the digits interchange their places

$\Rightarrow 10\text{ b}+\text{a}-18=10\text{a}+\text{b}$

$\Rightarrow 9b-9a=18$

$\Rightarrow b-a=2$

$\Rightarrow b=2+a$

Substituting in 1

$\Rightarrow a\times(2+a)=8$

$\Rightarrow a^2+2a-8=0$

$\Rightarrow a^2 + 4a - 2a - 8 = 0$
$\Rightarrow a(a + 4) - 2(a + 4) = 0$
$\Rightarrow (a - 2)(a + 4) = 0$
$\Rightarrow a = 2$
Thus, $b = 4$
Number is 42.

Q23. A two-digit number is such that the product of the digits is 12. When 36 is added to the number the digits interchange their places. Determine the number.

Solution:

Let the ones digit be ' a ' and tens digit be ' b '.
Given, two-digit number is such that the product of its digits is 12 .
$\Rightarrow ab = 12 - - - (1)$
Also, when 36 is added to the number, the digits interchange their places
$\Rightarrow 10b + a + 36 = 10a + b$
$\Rightarrow 9a - 9\,b = 36$
$\Rightarrow a - b = 4$
$\Rightarrow a = 4 + b$
Substituting in 1
$\Rightarrow b \times (4 + b) = 12$
$\Rightarrow b^2 + 4b - 12 = 0$
$\Rightarrow b^2 + 6\,b - 2\,b - 12 = 0$
$\Rightarrow b(b + 6) - 2(b + 6) = 0$
$\Rightarrow (b - 2)(b + 4) = 0$
$\Rightarrow b = 2$
Thus, $a = 6$
Number is 26.

Q24. A two-digit number is such that the product of the digits is 16. When 54 is subtracted from the number, the digits are interchanged. Find the number.

Solution:

Let the ones digit be ' a ' and tens digit be ' b '.
Given, two-digit number is such that the product of its digits is 16 .
$\Rightarrow ab = 16 - - (1)$
Also, when 54 is subtracted from the number, the digits interchange their places
$\Rightarrow 10\,b + a - 54 = 10a + b$
$\Rightarrow 9b - 9a = 54$
$\Rightarrow b - a = 6$
$\Rightarrow b = 6 + a$
Substituting in 1
$\Rightarrow a \times (6 + a) = 16$
$\Rightarrow a^2 + 6a - 16 = 0$

$\Rightarrow a^2 + 8a - 2a - 16 = 0$
$\Rightarrow a(a + 8) - 2(a + 8) = 0$
$\Rightarrow (a - 2)(a + 8) = 0$
$\Rightarrow a = 2$
Thus, $b = 8$
Number is 82.

Q25. Two numbers differ by 3 and their product is 504. Find the numbers.

Solution:

Given, two numbers differ by 3.
Let one of the numbers be 'a'.
Second number = a − 3
Also, their product is 504.
$\Rightarrow a(a - 3) = 504$
$\Rightarrow a^2 - 3a - 504 = 0$
$\Rightarrow a^2 - 24a + 21a - 504 = 0$
$\Rightarrow \mathrm{a}(\mathrm{a} - 24) + 21(\mathrm{a} - 24) = 0$
$\Rightarrow (a + 21)(a - 24) = 0$
$\Rightarrow a = -21, 24$
Thus numbers are $-21, -24$ or $24, 21$.

Q26. Two numbers differ by 4 and their product is 192. Find the numbers.

Solution:

Given, two numbers differ by 4.
Let one of the numbers be 'a'.
Second number = $a - 4$
Also, their product is 192.
$\Rightarrow a(a - 4) = 192$
$\Rightarrow a^2 - 4a - 192 = 0$
$\Rightarrow a^2 - 16a + 12a - 192 = 0$
$\Rightarrow a(a - 16) + 12(a - 16) = 0$
$\Rightarrow (a + 12)(a - 16) = 0$
$\Rightarrow a = -12, 16$
Thus numbers are $-12, -16$ or $12, 16$.

Q27. A two-digit number is 4 times the sum of its digits and twice the product of its digits. Find the number.

Solution:

Let the ones and tens digits be ' a ' and ' b ' respectively.
$10\,b + a = 4 \times (a + b)$
$\Rightarrow 6\,b = 3a$
$\Rightarrow a = 2\,b$

$Also, 10\,b + a = 2ab$

$\Rightarrow 10b + 2b = 2 \times 2b \times b$

$\Rightarrow 4b^2 = 12b$

$\Rightarrow b = 3$

$Thus, a = 6$

Number is 36 .

Q28. The difference of the squares of two positive integers is 180. The square of the smaller number is 8 times the larger number, find the numbers.

Solution:

Let the positive integers be ' a ' and ' b '.

Given, difference of the squares of two positive integers is 180 .

$\Rightarrow a^2 - b^2 = 180$

Also, square of the smaller number is 8 times the larger.

$\Rightarrow b^2 = 8\text{a}$

Thus, $a^2 - 8a - 180 = 0$

$\Rightarrow a^2 - 18a + 10a - 180 = 0$

$\Rightarrow a(a - 18) + 10(a - 18) = 0$

$\Rightarrow (a + 10)(a - 18) = 0$

$\Rightarrow a = -10,18$

Thus, the other number is

$324 - 180 = b^2$

$\Rightarrow b = 12$

Numbers are 12, 18.

Q29. The sum of two numbers is 18. The sum of their reciprocals is $\frac{1}{4}$. Find the numbers.

Solution:

Let the numbers be ' a ' and ' b '

Given, sum of two numbers is 18. The sum of their reciprocals is $\frac{1}{4}$

$\Rightarrow a + b = 18$

$\Rightarrow b = 18 - a$

Also, $\frac{1}{a} + \frac{1}{b} = \frac{1}{4}$

$$\Rightarrow \frac{1}{a} + \frac{1}{18 - a} = \frac{1}{4}$$

$\Rightarrow 18 \times 4 = 18a - a^2$

$\Rightarrow a^2 - 18a + 72 = 0$

$\Rightarrow a^2 - 12a - 6a + 72 = 0$

$\Rightarrow a(a - 12) - 6(a - 12) = 0$

$\Rightarrow (a - 6)(a - 12) = 0$

$\Rightarrow a = 6,12$

Numbers are 6,12 or 12,6.

Q30. The sum of two numbers a and b is 15, and the sum of their reciprocals $\frac{1}{a}$ and $\frac{1}{b}$ is $\frac{3}{10}$. Find the numbers a and b.

Solution:

Let the numbers be ' a ' and ' b '

Given, sum of two numbers is 15. The sum of their reciprocals is $\frac{1}{4}$

$\Rightarrow a+b=15$

$\Rightarrow b=15-a$

Also, $\frac{1}{a}+\frac{1}{b}=\frac{3}{10}$

$$\Rightarrow \frac{1}{a}+\frac{1}{15-a}=\frac{3}{10}$$

$\Rightarrow 15\times 10=45a-3a^2$

$\Rightarrow a^2-15a+50=0$

$\Rightarrow a^2-15a-5a+50=0$

$\Rightarrow a(a-10)-5(a-10)=0$

$\Rightarrow (a-5)(a-10)=0$

$\Rightarrow a=5,10$

Numbers are 5,10 or 10,5.

Q31. The sum of two numbers is 9. The sum of their reciprocals is $\frac{1}{2}$. Find the numbers.

Solution:

Let the numbers be 'a' and ' b '

Given, sum of two numbers is 18. The sum of their reciprocals is $\frac{1}{4}$

$\Rightarrow a+b=18$

$\Rightarrow b=18-a$

Also, $\frac{1}{a}+\frac{1}{b}=\frac{1}{4}$

$$\Rightarrow \frac{1}{a}+\frac{1}{18-a}=\frac{1}{4}$$

$\Rightarrow 18\times 4=18a-a^2$

$\Rightarrow a^2-18a+72=0$

$\Rightarrow a^2-12a-6a+72=0$

$\Rightarrow a(a-12)-6(a-12)=0$

$\Rightarrow (a-6)(a-12)=0$

$\Rightarrow a=6,12$

Numbers are 6,12 or 12,6.

Q32. Three consecutive positive integers are such that the sum of the square of the first and the product of other two is 46, find the integers.

Solution:

Let the three consecutive numbers be $a, a+1, a+2$
Given, there are three consecutive integers such that the sum of square of the first and the product of the other two is 46.
$\Rightarrow a^2+(a+1)(a+2)=46$
$\Rightarrow 2a^2+3a+2=46$
$\Rightarrow 2a^2+3a-44=0$
$\Rightarrow 2a^2+11a-8a-44=0$
$\Rightarrow a(2a+11)-4(2a+11)=0$
$\Rightarrow (a-4)(2a+11)=0$
Thus, $a=4$
Numbers are 4, 5, 6.

Q33. The difference of squares of two numbers is 88. If the larger number is 5 less than twice the smaller number, then find the two numbers.

Solution:

Let the numbers be 'a' and ' b '.
Given, difference of squares of two numbers is 88 .
$\Rightarrow a^2-b^2=88$
Also, the larger number is 5 less than twice the smaller number.
$\Rightarrow \mathrm{a}=2\mathrm{~b}-5$
Thus, $(2b-5)^2-b^2=88$
$\Rightarrow 4b^2+25-20b-b^2=88$
$\Rightarrow 3b^2-20b-63=0$
$\Rightarrow 3b^2-27b+7b-63=0$
$\Rightarrow 3b(b-9)+7(b-9)=0$
$\Rightarrow (3b+7)(b-9)=0$
$\Rightarrow b=9$
Thus, $a=2\times 9-5=13$.

Q34. The difference of squares of two numbers is 180. The square of the smaller number is 8 times the larger number. Find two numbers.

Solution:

Let the numbers be 'a' and ' b '.
Given, difference of the squares of two numbers is 180 .
$\Rightarrow a^2-b^2=180$
Also, square of the smaller number is 8 times the larger.
$\Rightarrow b^2=8a$
Thus, $a^2-8a-180=0$
$\Rightarrow a^2-18a+10a-180=0$
$\Rightarrow a(a-18)+10(a-18)=0$
$\Rightarrow (a+10)(a-18)=0$
$\Rightarrow a=-10,18$

Thus, the other number is
$324-180=b^2$
$\Rightarrow b=12$
Numbers are 12,18 or $-12,18$.

Q35. Find two consecutive odd positive integers, sum of whose squares is 970.

Solution:

Let the consecutive odd positive integers be ' a ' and $a+2$
$\Rightarrow a^2+(a+2)^2=970$
$\Rightarrow 2a^2+4a-966=0$
$\Rightarrow a^2+2a-483=0$
$\Rightarrow a^2+23a-21a-483=0$
$\Rightarrow a(a+23)-21(a+23)=0$
$\Rightarrow (a-21)(a+23)=0$
Thus, $a=21$
Consecutive odd positive integers are 21, 23.

Q36. The difference of two natural numbers is 3 and the difference of their reciprocals is $\frac{3}{28}$. Find the numbers.

Solution:

Let the natural numbers be ' a ' and ' b '.
Given, difference of two natural numbers is 3 and difference of their reciprocals is $\frac{3}{28}$
$\Rightarrow a-b=3$
$\Rightarrow a=b+3$
and $\frac{1}{b}-\frac{1}{a}=\frac{3}{28}$
$$\Rightarrow \frac{1}{b}-\frac{1}{b+3}=\frac{3}{28}$$
$\Rightarrow 28(b-b-3)=-3(b^2+3b)$
$\Rightarrow b^2+3b-28=0$
$\Rightarrow b^2+7b-4b-28=0$
$\Rightarrow b(b+7)-4(b+7)=0$
$\Rightarrow (b-4)(b+7)=0$
$\Rightarrow b=4$
Numbers are, 4, 7

Q37. The sum of the squares of two consecutive odd numbers is 394 . Find the numbers.

Solution:

Given: The sum of the squares of two consecutive odd numbers is 394 .
To find: the numbers.
Solution: Let the consecutive odd number be 'a' and a + 2According to given condition,

$a^2 + (a+2)^2 = 394$ Use the formula $(x+y)^2 = x^2 + y^2 + 2xy$ in $(a+2)^2$ Here $x = a$ and $y = 2, \Rightarrow a^2 + a^2 + 4 + 4a = 394$
$\Rightarrow 2a^2 + 4a + 4 - 394 = 0 \Rightarrow 2a^2 + 4a - 390 = 0$Take 2 common out of the above equation,
$\Rightarrow \mathrm{a}^2 + 2\mathrm{a} - 195 = 0$ Factorise by splitting the middle term.
$\Rightarrow a^2 + 15a - 13a - 195 = 0$
$\Rightarrow a(a+15) - 13(a+15) = 0$
$\Rightarrow (a-13)(a+15) = 0$
Thus, $\mathrm{a} = 13, -15$When$\mathrm{a} = 13$ then $\mathrm{a} + 2 = 15$ And when $a = -15$ then $a + 2 = -13$
So Consecutive odd numbers are 13, 15 and -15,-13.

Q38. The sum of the squares of two consecutive multiples of 7 is 637. Find the multiples.

Solution:

Let the consecutive multiples of 7 be *'a'* and $a + 7$
$\Rightarrow a^2 + (a+7)^2 = 637$
$\Rightarrow 2a^2 + 14a - 588 = 0$
$\Rightarrow 2a^2 + 42a - 28a - 588 = 0$
$\Rightarrow 2a(a+21) - 28(a+21) = 0$
$\Rightarrow (2a-28)(a+21) = 0$
Thus, $\mathrm{a} = 14$
Consecutive multiples of 7 are 14, 21.

Q39. The sum of the squares of two consecutive even numbers is 340. Find the numbers.

Solution:

Let the consecutive even integers be 'a' and a +2
$\Rightarrow a^2 + (a+2)^2 = 340$
$\Rightarrow 2a^2 + 4a - 336 = 0$
$\Rightarrow a^2 + 2a - 168 = 0$
$\Rightarrow a^2 + 14a - 12a - 168 = 0$
$\Rightarrow a(a+14) - 12(a+14) = 0$
$\Rightarrow (a-12)(a+14) = 0$
Thus, $a = 12$ or -14
Consecutive even integers are 12,14 or $-14, -12$.

Q40. The numerator of a fraction is 3 less than the denominator. If 2 is added to both the numerator and the denominator, then the sum of the new fraction and the original fraction is $\frac{29}{20}$, find the original fraction.

Solution:

Let the denominator be *'a'*.
Numerator $= a - 3$
[As numerator is 3 less than denominator]Now, if 2 is added to both the numerator

and the denominator, then the sum of the new fraction and the original fraction is $\frac{29}{20}$.

$\Rightarrow \frac{a-3}{a}+\frac{a-3+2}{a+2}=\frac{29}{20}$

$\Rightarrow (a-3)(a+2)+(a-1)(a)=\frac{29a(a+2)}{20}$

$\Rightarrow 20(a^2-a-6)+20a^2-20a=29a^2+58a$

$\Rightarrow 11a^2-98a-120=0$

$\Rightarrow 11a^2-110a+12a-120=0$

$\Rightarrow 11\mathrm{a}(\mathrm{a}-10)+12(\mathrm{a}-10)=0$

$\Rightarrow (11a+12)(a-10)=0$

$\Rightarrow \mathrm{a}=10$ or $\mathrm{a}=-\frac{12}{11}$ Since, denominator can't be a fraction $\Rightarrow \mathrm{a}=10$

and numerator $=a-3=10-3=7$

Thus the original fraction is $\frac{7}{10}$.

Exercise 8.8

Q1. The speed of a boat in still water is 8 km/hr. It can go 15 km upstream and 22 km downstream in 5 hours. Find the speed of the stream.

Solution:

Given: The speed of a boat in still water is 8 km/hr. It can go 15 km upstream and 22 km downstream in 5 hours.

To find: the speed of the stream.

Solution: Let the speed of stream be $'a'$km/hr.

Given, speed of a boat in still water is 8 km/hr. It can go 15 km upstream and 22 km downstream in 5 hours.

Going upstream means that boat is going in opposite direction of the stream so speeds will be added and going downstream means that the boat is going in the same direction of the stream. So,

Relative speed of boat going upstream $=8-a$

Relative speed of boat going downstream $=8+a$

Time = distance/speedTotal time is given to be 5 hrs .

$\Rightarrow \frac{15}{8-a}+\frac{22}{8+a}=5$

$\Rightarrow \frac{15(8+a)+22(8-a)}{(8-a)(8+a)}=5$

$\Rightarrow 15(8+a)+22(8-a)=5(8-a)(8+a)$ Apply the formula $(a-b)(a+b)=a^2-b^2$ in $(8-a)(8+a)$ Here $a=8$ and $b=a$.

$\Rightarrow 120+15\mathrm{a}+176-22\mathrm{a}=5(64-\mathrm{a}^2) \Rightarrow 296-7\mathrm{a}=-5\mathrm{a}^2+320$

$\Rightarrow 5a^2-7a-24=0$ Factorize the equation by splitting the middle term

$\Rightarrow 5a^2-15a+8a-24=0$

$\Rightarrow 5a(a-3)+8(a-3)=0$
$\Rightarrow (5a+8)(a-3)=0 \Rightarrow (5a+8)=0$ and $(a-3)=0$
$\Rightarrow a=\frac{-8}{5}$ and $a=3$
Since $a=\frac{-8}{5}$ is not possible
$\Rightarrow a=3$ km/hrHence speed of the stream is 3 km/hr.

Q2. A passenger train takes 3 hours less for a journey of 360 km, if its speed is increased by 10 km/hr from its usual speed. What is the usual speed?

Solution:

Distance = speed × time
Given, passenger train takes 3 hours less for a journey of 360 km, if its speed is increased by 10 km/hr from its usual speed.
Let the speed be ' s ' and time be ' t '.
$\Rightarrow \mathrm{st}=360$
$\Rightarrow t=360/\mathrm{s}$
Also, $360=(s+10)(t-3)$
$\Rightarrow 360=(s+10)\left(\frac{360}{s}-3\right)$
$\Rightarrow 360s=360s+3600-3s^2-30s$
$\Rightarrow s^2+10s-1200=0$
$\Rightarrow s^2+40s-30s-1200=0$
$\Rightarrow \mathrm{s}(\mathrm{s}+40)-30(\mathrm{s}+40)=0$
$\Rightarrow (\mathrm{s}-30)(\mathrm{s}+40)=0$
$\Rightarrow \mathrm{s}=30$ km/hr.

Q3. A fast train takes one hour less than a slow train for a journey of 200 km . If the speed of the slow train is 10 km/hr less than that of the fast rain, find the speed of the two trains.

Solution:

$Speed=\frac{distance}{time}$
Let the speed of the faster train be 'a' km/hr.
Speed of the slow train $=a-10$ km/hr
Also, fast train takes one hour less than a slow train for a journey of 200 km .
$\Rightarrow \frac{200}{a-10}-\frac{200}{a}=1$
$\Rightarrow 200a+2000-200a=a^2-10a$
$\Rightarrow a^2-10a-2000=0$
$\Rightarrow a^2-50a+40a-2000=0$
$\Rightarrow a(a-50)+40(a-50)=0$
$\Rightarrow (a+40)(a-50)=0$
$\Rightarrow a=50$ km/hr

Speed of the trains is 50 km/hr, 40 km/hr

Q4. A passenger train takes one hour less for a journey of 150 km if its speed is increased by 5 km/hr from the usual speed. Find the usual speed of the train.

Solution:

$Time\ = \frac{distance}{speed}$

Let the speed of the train be 'a' km/hr.

Given, passenger train takes one hour less for a journey of 150 km if its speed is increased by

5 km/hr from the usual speed.

$\Rightarrow \frac{150}{a} - \frac{150}{a+5} = 1$

$\Rightarrow 150(a+5-a) = a^2 + 5a$

$\Rightarrow a^2 + 5a - 750 = 0$

$\Rightarrow a^2 + 30a - 25a - 750 = 0$

$\Rightarrow a(a+30) - 25(a+30) = 0$

$\Rightarrow (a-25)(a+30) = 0$

$\Rightarrow a = 25$ km/hr

Q5. The time taken by a person to cover 150 km was 2.5 hrs more than the time taken in the return journey. If he returned at a speed of 10 km/hr more than the speed of going, what was the speed per hour in each direction?

Solution:

$Time\ = \frac{distance}{speed}$

Let the speed of person on onward journey be $'\ a\ '$ km/hr

Speed at which he returned $= a - 10$ km/hr

Given, time taken by a person to cover 150 km was 2.5 hrs more than the time taken in the return journey.

$\Rightarrow \frac{150}{a-10} - \frac{150}{a} = 2.5$

$\Rightarrow 150(a - a + 10) = 2.5a(a-10)$

$\Rightarrow 1500 = 2.5\text{a}^2 - 25\text{a}$

$\Rightarrow a^2 - 10a - 600 = 0$

$\Rightarrow a^2 - 30a + 20a - 600 = 0$

$\Rightarrow a(a-30) + 20(a-30) = 0$

$\Rightarrow (a+20)(a-30) = 0$

$\Rightarrow a = 30$ km/hr.

Q6. A plane left 40 minutes late due to bad weather and in order to reach its destination, 1600 km away in time, it had to increase its speed by 400 km/hr from its usual speed. Find the usual speed of the plane.

Solution:

$Time = \frac{distance}{speed}$

Given, plane left 40 minutes late due to bad weather and in order to reach its destination, 1600 km away in time, it had to increase its speed by 400 km/hr from its usual speed.

Let the usual speed be ' a '.

$$\frac{1600}{a} - \frac{1600}{a+400} = \frac{40}{60}$$

$\Rightarrow 3(1600 \times 400) = 2(a^2 + 400a)$

$\Rightarrow a^2 + 400a - 960000 = 0$

$\Rightarrow a^2 + 1200a - 800a - 960000 = 0$

$\Rightarrow a(a+1200) - 800(a+1200) = 0$

$\Rightarrow (a+1200)(a-800) = 0$

$\Rightarrow a = 800 \text{ km/hr}$

Q7. An airplane takes 1 hour less for a journey of 1200 km if its speed is increased by 100 km/hr from its usual speed. Find its usual speed.

Solution:

$Time = \frac{distance}{speed}$

Given, airplane takes 1 hour less for a journey of 1200 km if its speed is increased by 100 km/hr from its usual speed.

Let the usual speed be ' a '.

$$\frac{1200}{a} - \frac{1200}{a+100} = 1$$

$\Rightarrow 1200 \times (a + 100 - a) = a^2 + 100a$

$\Rightarrow a^2 + 100a - 120000 = 0$

$\Rightarrow a^2 + 400a - 300a - 120000 = 0$

$\Rightarrow a(a+400) - 300(a+400) = 0$

$\Rightarrow (a+400)(a-300) = 0$

$\Rightarrow a = 300\ km/hr$

Q8. A passenger train takes 2 hours less for a journey of 300 km if its speed is increased by 5 km/hr from its usual speed. Find the usual speed of the train.

Solution:

Given, passenger train takes 2 hours less for a journey of 300 km if its speed is increased by 5 km/hr from its usual speed.

Let the usual speed be ' a ' km/hr

Increased speed $= (a+5)$km/ hr

we know, time $= \frac{\text{distance}}{\text{speed}}$

$\Rightarrow$ time taken by train to cover with usual speed to travel 300 km $= \frac{300}{a}$ km/h

$\Rightarrow$ time taken by train to cover with increased speed to travel 300 km $= \frac{300}{a+5}$ km/h

Therefore, According to question

$$\Rightarrow \frac{300}{a} - \frac{300}{a+5} = 2$$
$$\Rightarrow 300 \times (a+5-a) = 2(a^2+5a)$$
$$\Rightarrow a^2 + 5a - 750 = 0$$
$$\Rightarrow a^2 + 30a - 25a - 750 = 0$$
$$\Rightarrow a(a+30) - 25(a+30) = 0$$
$$\Rightarrow (a+30)(a-25) = 0$$
$$\Rightarrow a = 25\ km/hr$$

Q9. A train covers a distance of 90 km at a uniform speed. Had the speed been 15 km/hour more, it would have taken 30 minutes less for the journey. Find the original speed of the train.

Solution:

$$Time\ = \frac{distance}{speed}$$

Given, train covers a distance of 90 km at a uniform speed. Had the speed been 15 km/hour more, it would have taken 30 minutes less for the journey.

Let the usual speed be ' a '.

$$\frac{90}{a} - \frac{90}{a+15} = \frac{30}{60}$$
$$\Rightarrow 90 \times (a+15-a) = \frac{(a^2+15a)}{2}$$
$$\Rightarrow a^2 + 15a - 2700 = 0$$
$$\Rightarrow a^2 + 60a - 45a - 2700 = 0$$
$$\Rightarrow a(a+60) - 45(a+60) = 0$$
$$\Rightarrow (a+60)(a-45) = 0$$
$$\Rightarrow a = 45\ km/hr$$

Q10. A train travels 360 km at a uniform speed. If the speed had been 5 km/hr more, it would have taken 1 hour less for the same journey. Find the speed of the train.

Solution:

To find: Speed of the train

Method 1: Let the speed of the train be x km/hr.

Time taken to cover 360 km $= \frac{360}{x}$ hr,

As

$$\text{time} = \frac{\text{distance}}{\text{speed}}$$

Now, given that if the speed would be 5 km/hr more, the same distance would be covered in 1 hour less, i.e.if speed $= x + 5$, and

$$\text{time} = \left(\frac{360}{x} - 1\right) \text{ hours}$$

then, using distance = speed × time, we have

$$(x+5)\left(\frac{360}{x}-1\right)=360$$

Now we can form the quadratic equation from this equation

$$360-x+\frac{1800}{x}-5=360$$

$$\frac{360x-x^2+1800-5x}{x}=360$$

Now, cross multiplying we get

$$360x-x^2+1800-5x=360x$$

$$x^2+5x-1800=0$$

Now we have to factorize in such a way that the product of the two numbers is 1800 and the difference is $5x^2+45x-40\text{x}-1800=0$

$$x(x+45)-40(x+45)=0$$

$$(x+45)(x-40)=0$$

$$x=-45,40$$

since, the speed of train can't be negative,so, speed will be 40 km/hour.

Q11. An express train takes 1 hour less than a passenger train to travel 132 km between Mysore and Bangalore (without taking into consideration the time they stop at intermediate stations). If the average speed of the express trains is 11 km/hr more than that of the passenger train, find the average speeds of the two trains.

Solution:

$$Time\ =\frac{distance}{speed}$$

Now, express train takes 1 hour less than a passenger train to travel 132 km between Mysore and Bangalore. The average speed of the express trains is 11 km/hr more than that of the passenger train.

Let the average speed of passenger train be ' a '.

$$\frac{132}{a}-\frac{132}{a+11}=1$$

$$\Rightarrow 132\times(a+11-a)=a^2+11a$$

$$\Rightarrow a^2+11a-1452=0$$

$$\Rightarrow a^2+44a-33a-1452=0$$

$$\Rightarrow a(a+44)-33(a+44)=0$$

$$\Rightarrow (a+44)(a-33)=0$$

$$\Rightarrow a=33\text{ km/hr}$$

Speed of express train = 44 km/hr.

Q12. An aeroplane left 50 minutes later than its scheduled time, and in order to reach the destination, 1250 km away, in time, it had to increase its speed by 250 km/hr from its usual speed. Find its usual speed.

Solution:

Given: An airplane left 50 minutes later than its scheduled time, and in order to reach the destination, 1250 km away, in time, it had to increase its speed by 250 km/hr from its usual speed.
To find: its usual speed.
Given, airplane left 50 minutes later than its schedule time, and in order to reach the destination, 1250 km away, in time, it had to increase its speed to 250 km/hr from its usual speed.
$Time \ = \frac{distance}{speed}$
Let the usual speed be ' a '.
Since the speed of the aeroplane has been increased the time taken by the now will be less. So we will subtract the time taken by the aeroplane when the speed has been increased from the time it would have taken originally and this time is equal to 50 min. Since the speed has been given in $\frac{\mathrm{km}}{\mathrm{h}}$ so we need to convert 50 min into hours. To convert 50 min into hours divide it by $60 \mathrm{as} 1 \mathrm{hr}=60 \mathrm{~min}$
$\frac{1250}{a}-\frac{1250}{a+250}=\frac{50}{60}$ Take 1250 common from the LHS,So, $\Rightarrow 1250\left(\frac{1}{a}-\frac{1}{a+250}\right)=\frac{5}{6}$
$\Rightarrow 1250\left(\frac{a+250-a}{a(a+250)}\right)=\frac{5}{6} \Rightarrow(6 \times 1250)(a+250-a)=5 a(a+250)$
$\Rightarrow(6 \times 1250)(a+250-a)=5\left(a^2+250 a\right) \Rightarrow 1875000=5 a^2+1250 a \Rightarrow 5 a^2+$ $1250 a-1875000=0$ Take 5 common out of the above equation,
$\Rightarrow a^2+250 a-375000=0$ Factorise the equation by splitting the middle term as:
$\Rightarrow a^2+750 a-500 a-375000=0$
$\Rightarrow a(a+750)-500(a+750)=0$
$\Rightarrow(a+750)(a-500)=0 \Rightarrow(a+750)=0$ and $(a-500)=0$
$\Rightarrow a=-750$ $and\, a=500$ Since the speed cannot be negative.
$\Rightarrow a=\frac{500 \, km}{\mathrm{hr}}$
Hence the speed of aeroplane is 500 km/hr.

Q13. While boarding an aeroplane, a passenger got hurt. The pilot showing promptness and concern, made arrangements to hospitalize the injured and so the plane started late by 30 minutes to reach the destination, 1500 km away in time, the pilot increased the speed by 100 km/hr. Find the original speed / hour of the plane.

Solution:

$Time \ = \frac{distance}{speed}$

Given, while boarding an aeroplane, a passenger got hurt and the plane started late by 30 minutes to reach the destination, 1500 km away in time, the pilot increased the speed by 100 km/hr.
Let the usual speed be ' a '.
$\frac{1500}{a}-\frac{1500}{a+100}=\frac{30}{60}$

$\Rightarrow 2 \times 1500 \times (a + 100 - a) = a^2 + 100a$
$\Rightarrow a^2 + 100a - 300000 = 0$
$\Rightarrow a^2 + 600a - 500a - 300000 = 0$
$\Rightarrow a(a + 600) - 500(a + 600) = 0$
$\Rightarrow (a + 750)(a - 500) = 0$
$\Rightarrow a = 500\ km/hr$

Q14. A motorboat whose speed in still water is 18 km/hr takes 1 hour more to go 24 km upstream that to return downstream to the same spot. Find the speed of the stream.

Solution:

$$Time\ = \frac{Distance}{speed}$$

Given, motorboat whose speed in still water is 18 km/hr takes 1 hour more to go 24 km upstream that to return downstream to the same spot.
Let the speed of stream be $'a'$km/hr.
Relative speed of boat going upstream $= 18 - a$ km/hr
Relative speed of boat going downstream $= 18 + a$ km/hr

$$\Rightarrow \frac{24}{18 - a} - \frac{24}{18 + a} = 1$$

$\Rightarrow 24(18 + a - 18 + a) = -a^2 + 324$
$\Rightarrow a^2 + 48a - 324 = 0$
$\Rightarrow a^2 + 54a - 6a - 324 = 0$
$\Rightarrow a(a + 54) - 6(a + 54) = 0$
$\Rightarrow (a - 6)(a + 54) = 0$
$\Rightarrow a = 6$ km/hr

Exercise 8.9

Q1. Ashu is x years old while his mother Mrs. Veena is x^2 years old. Five years hence Mrs. Veena will be three times old as Ashu. Find their present ages.

Solution:

Given, Ashu is x years old while his mother Mrs. Veena is x^2 years old.
After 5 years, Mrs. Veena will be three times old as Ashu.
$\Rightarrow x^2 + 5 = 3(x + 5)$
$\Rightarrow x^2 - 3x - 10 = 0$
$\Rightarrow x^2 - 5x + 2x - 10 = 0$
$\Rightarrow x(x - 5) + 2(x - 5) = 0$
$\Rightarrow (x + 2)(x - 5) = 0$
$\Rightarrow x = 5\ km/hr$

Q2. The sum of the ages of a man and his son is 45 years. Five years ago, the product of their ages was four times the man's age at the time. Find their present ages.

Solution:

Let the present ages of the man and son be ' a ' and ' b ' respectively.

Given, sum of the ages of a man and his son is 45 years.

$\Rightarrow a+b=45$

Ans, five years ago, the product of their ages was four times the man's age at the time.

$\Rightarrow(a-5)(b-5)=4(a-5)$

$\Rightarrow b-5=4$

$\Rightarrow b=9$ years

Thus, $a=45-9=36$ years

Q3. The product of Shikha's age five years ago and her age 8 years later is 30 , her age at both times being given in years. Find her present age.

Solution:

Let the present age of Shikha be 'a' years.

Given, product of Shikha's age five years ago and her age 8 years later is 30

$\Rightarrow(a-5)(a+8)=30$

$\Rightarrow a^2+3a-40-30=0$

$\Rightarrow a^2+10a-7a-70=0$

$\Rightarrow a(a+10)-7(a+10)=0$

$\Rightarrow(a-7)(a+10)=0$

$\Rightarrow a=7$ years

Q4. The product of Ramu's age (in years) five years ago and his age (in years) nine years later is 15. Determine Ramu's present age.

Solution:

Let the present age of Ramu be $'a'$ years.

Given, product of Ramu's age (in years) five years ago and his age (in years) nine years later is 15.

$\Rightarrow(a-5)(a+9)=15$

$\Rightarrow a^2+4a-45-15=0$

$\Rightarrow a^2+10a-6a-60=0$

$\Rightarrow a(a+10)-6(a+10)=0$

$\Rightarrow(a-6)(a+10)=0$

$\Rightarrow a=6$ years.

Q5. Is the following situation possible? If so, determine their present ages.
The sum of the ages of two friends is 20 years. Four years ago, the product of their ages in years was 48.

Solution:

Given, sum of the ages of two friends is 20 years. Four years ago, the product of their ages in years was 48 Let the age of one of the friends be 'a'

Age of the other friend $= 20 - \mathrm{a}$
$\Rightarrow (a-4)(20-a-4)=48$
$\Rightarrow (a-4)(16-a)=48$
$\Rightarrow a^2-20a+64+48=0$
$\Rightarrow a^2-20a+112=0$
$D=b^2-4ac$
$\Rightarrow D=400-4\times 112=-48$
Thus, roots are not real as $\mathrm{D}<0$
The following situation is not possible.

Q6. A girl is twice as old as her sister. Four years hence, the product of their ages (in years) will be 160. Find their present ages.

Solution:

Let the present ages of the younger sister be ' a '.
Given, girl is twice as old as her sister.
Age of elder sister $= 2a$
Also, four years ago, the product of their ages (in years) will be 160.
$\Rightarrow (a+4)(2a+4)=160$
$\Rightarrow 2\mathrm{a}^2+12\mathrm{a}+16-160=0$
$\Rightarrow a^2+6a-72=0$
$\Rightarrow a^2+12a-6a-72=0$
$\Rightarrow a(a+12)-6(a+12)=0$
$\Rightarrow (a-6)(a+12)=0$
$\Rightarrow a=6$ years
Age of sisters 6 years and 12 years.

Q7. The sum of the reciprocals of Rehman's ages (in years) 3 years ago and 5 years from now is $\frac{1}{3}$. Find the present age.

Solution:

Let Rehman's present age be ' a '.
Given, sum of the reciprocals of Rehman's ages (in years) 3 years ago and 5 years from now is $\frac{1}{3}$.

$$\frac{1}{a-3}+\frac{1}{a+5}=\frac{1}{3}$$

$\Rightarrow 3(a-3+a+5)=a^2+2a-15$
$\Rightarrow a^2-4a-21=0$
$\Rightarrow a^2-7a+3a-21=0$
$\Rightarrow a(a-7)+3(a-7)=0$
$\Rightarrow (a+3)(a-7)=0$
$\Rightarrow a=7$

Exercise 8.10

Q1. The hypotenuse of a right triangle is 25 cm. The difference between the lengths of the other two sides of the triangle is 5 cm. Find the lengths of these sides.

Solution:

Given: The hypotenuse of a right triangle is 25 cm. The difference between the lengths of the other two sides of the triangle is 5 cm.

To find: the lengths of these sides.

Solution: We know (Hypotenuse) $)^2 = ($ perpendicular) $)^2 + ($ base $)^2$

Given, hypotenuse of a right triangle is 25 cm. The difference between the lengths of the other two sides of the triangle is 5 cm

Let the base be $'b'$.

$\Rightarrow$ Perpendicular $= b - 5$ Put the known values in (1),

$\Rightarrow 25^2 = b^2 + (b-5)^2$

Apply the formula $(x-y)^2 = x^2 + y^2 - 2xy$ in $(b-5)^2$.

Here $x = b$ and $y = 5$

$\Rightarrow 625 = b^2 + b^2 + 25 - 10b \Rightarrow 625 = 2b^2 + 25 - 10b$

$\Rightarrow 2b^2 + 25 - 10b = 625 \Rightarrow 2b^2 + 25 - 10b - 625 = 0 \Rightarrow 2b^2 - 10b - 600 = 0$

Take out 2 common of the above equation.

$\Rightarrow b^2 - 5b - 300 = 0$

Factorise the above quadratic equation by splitting the middle term.

$\Rightarrow b^2 - 20b + 15b - 300 = 0$

$\Rightarrow b(b-20) + 15(b-20) = 0$

$\Rightarrow (b-20)(b+15) = 0 \Rightarrow (b-20) = 0$ and $(b+15) = 0$

$\Rightarrow b = 20$ and $b = -15$ Since the length of any side cannot be negative, We will ignore -15.

$\Rightarrow b = 20$

Perpendicular $= 20 - 5 = 15$

Hence, Sides are 15 and 20.

Q2. The hypotenuse of a right triangle is $3\sqrt{10}$ cm. If the smaller leg is tripled and the longer leg doubled, new hypotenuse will be $9\sqrt{5}$ cm. How long are the legs of the triangle?

Solution:

Let the smaller leg be ' a ' and longer leg be ' b '.

$Hypotenus^2 = length^2 + breadth^2$

Given, hypotenuse of a right triangle is $3\sqrt{10}$ cm

$\Rightarrow 9 \times 10 = a^2 + b^2$

$\Rightarrow a^2 + b^2 = 90 - - - - - (1)$

Now, the smaller leg is tripled and the longer leg doubled, new hypotenuse is $9\sqrt{5}$ cm.

$\Rightarrow (3a)^2 + (2b)^2 = 81 \times 5$

$\Rightarrow 9a^2 + 4b^2 = 405$

Multiplying (1) by 4 and subtracting from eq 2
$\Rightarrow 5a^2 = 45$
$\Rightarrow a^2 = 9$
$\Rightarrow a = 3$
Thus, $9 + b^2 = 90$
$\Rightarrow b^2 = 81$
$\Rightarrow b = 9$.

Q3. A pole has to be erected at a point on the boundary of a circular park of diameter 13 metres in such a way that the difference of its distances from two diametrically opposite fixed gates A and B on the boundary is 7 metres. Is it the possible to do so? If yes, at what distances from the two gates should the pole be erected?

Solution:

Let the distance of pole from gate A be ' a '.
Difference of the distance of the pole from two diametrically opposite fixed gates A and B on the boundary is 7 metres.
Distance of pole from gate $B = a - 7m$
Diameter of the park = 13 m
$Hypotenuse^2 = length^2 + breadth^2$
$\Rightarrow 13^2 = a^2 + (a - 7)^2$
$\Rightarrow 169 = 2a^2 + 49 - 14a$
$\Rightarrow a^2 - 7a - 60 = 0$
$\Rightarrow a^2 - 12a + 5a - 60 = 0$
$\Rightarrow a(a - 12) + 5(a - 12) = 0$
$\Rightarrow (a + 5)(a - 12) = 0$
$\Rightarrow a = 12\ m$
Thus distance of pole is 12 m from gate A and 5 m from gate B.

Q4. The diagonal of a rectangular field is 60 metres more than the shorter side. If the longer side is 30 metres more than the shorter side, find the sides of the field

Solution:

Let the shorter side be ' a '.
Given, diagonal of a rectangular field is 60 metres more than the shorter side
Diagonal $= a + 60$
Also, longer side is 30 metres more than the shorter side
Longer side $= a + 30$
$Hypotenuse^2 = Length^2 + Breadth^2$
$\Rightarrow (a + 60)^2 = (a + 30)^2 + a^2$
$\Rightarrow a^2 + 120a + 3600 = a^2 + 60a + 900 + a^2$
$\Rightarrow a^2 - 60a - 2700 = 0$
$\Rightarrow a^2 - 90a + 30a - 2700 = 0$
$\Rightarrow a(a - 90) + 30(a - 90) = 0$

$\Rightarrow (a+30)(a-90)=0$
$\Rightarrow a=90 \mathrm{~m}$
Length of sides $=90 \mathrm{~m}, 120 \mathrm{~m}$

Exercise 8.11

Q1. The perimeter of a rectangular field is 82 m and its area is $400 \mathrm{~m}^2$. Find the breadth of the rectangle.

Solution:

Perimeter of a rectangle $=2(l+b)$
Area of the rectangle $=l \times b$
Given, perimeter of a rectangular field is 82 m and its area is $400 \mathrm{~m}^2$
Let the breadth be ' a ' m and length be ' b ' m
$\Rightarrow 2(a+b)=82$
$\Rightarrow b=41-a$
Also, $a \times b=400$
$\Rightarrow a \times(41-a)=400$
$\Rightarrow a^2-41 a+400=0$
$\Rightarrow a^2-25 a-16 a+400=0$
$\Rightarrow a(a-25)-16(a-25)=0$
$\Rightarrow (a-16)(a-25)=0$
$\Rightarrow a=16,25$
Assuming breadth to smaller, thus breadth $=16 \mathrm{~m}$.

Q2. The length of a hall is 5 m more than its breadth. If the area of the floor of the hall is $84 \mathrm{~m}^2$, what are the length and breadth of the hall?

Solution:

Let the breadth of the hall be ' a '
Length $=a+5$
Given, area of the floor of the hall is $84 \mathrm{~m}^2$
$\Rightarrow a(a+5)=84$
$\Rightarrow a^2+5 a-84=0$
$\Rightarrow a^2+12 a-7 a-84=0$
$\Rightarrow a(a+12)-7(a+12)=0$
$\Rightarrow (a-7)(a+12)=0$
$\Rightarrow a=7 \mathrm{~m}$
Length of the hall $=7+5=12 \mathrm{~m}$.

Q3. Two squares have sides x cm and $(x+4)$cm. The sum of their areas is $656 \mathrm{~cm}^2$. Find the sides of the squares.

Solution:

Area of a square = side × side
Given, squares have sides x cm and $(x+4)$cm. The sum of their areas is 656 cm^2.
$\Rightarrow x^2+(x+4)^2=656$
$\Rightarrow x^2+x^2+8x+16=656$
$\Rightarrow x^2+4x-320=0$
$\Rightarrow x^2+20x-16x-320=0$
$\Rightarrow x(x+20)-16(x+20)=0$
$\Rightarrow (x-16)(x+20)=0$
$\Rightarrow x=16\ cm$
The other side $=16+4=20$ cm.

Q4. The area of a right-angled triangle is 165 m^2. Determine its base and altitude if the latter exceeds the former by 7 m.

Solution:

Let the base of the triangle be ' a '.
Given, altitude exceeds the base by 7 m .
$\Rightarrow$ Altitude $=a+7$ m
Area of a right angled triangle $=\frac{1}{2}\times$ base $\times$ height
$$\Rightarrow \frac{1}{2}\times a(a+7)=165$$
$\Rightarrow a^2+7a-330=0$
$\Rightarrow a^2+22a-15a-330=0$
$\Rightarrow a(a+22)-15(a+22)=0$
$\Rightarrow (a-15)(a+22)=0$
$\Rightarrow a=15\ m$
Altitude $=15+7=22$ m

Q5. Is it possible to design a rectangular mango grove whose length is twice its breadth and the area is 800 m^2 ? If so, find its length and breadth.

Solution:

Let the breadth be ' a ' m
Given, rectangular mango grove whose length is twice its breadth.
Length $=2a$
Area $=800\ m^2$
$\Rightarrow 2a\times a=800$
$\Rightarrow a^2=400$
$\Rightarrow a=20$ m
Thus, length $=40$ m.

Q6. Is it possible to design a rectangular park of perimeter 80 m and area 400 m^2 ? If so, find its length and breadth.

Solution:

Let the length and breadth of park be $'a'$ and $'b'$ m
Given, rectangular park of perimeter 80 m and area 400 m^2.
$\Rightarrow 2(a+b)=80$
$\Rightarrow a+b=40$
$\Rightarrow a=40-b$
$\Rightarrow a\times b=400$
$\Rightarrow (40-b)b=400$
$\Rightarrow b^2-40b+400=0$
$\Rightarrow (b-20)^2=0$
$\Rightarrow b=20\ m$
Thus, it is possible to design a rectangular park with length $=20$ m and breadth $=$ 20 m.

Q7. Sum of the areas of two squares is 640 m^2. If the difference of their perimeters is 64 m, find the sides of the two squares.

Solution:

Area of a square $=s^2$
Perimeter of a square $=4$ s
Let the sides of the square be a and b respectively.
Given, sum of the areas of two squares is 640 m^2 and the difference of their perimeters is 64 m .
$\Rightarrow a^2+b^2=640$ and
$4a-4b=64$
$\Rightarrow a-b=16$
$\Rightarrow a=16+b$
$\Rightarrow (16+b)^2+b^2=640$
$\Rightarrow 2b^2+32b+256=640$
$\Rightarrow b^2+16b-192=0$
$\Rightarrow b^2+24b-16b-192=0$
$\Rightarrow (b-8)(b+24)=0$
$\Rightarrow b=8\ m$
Thus, $a=24$ m.

Q8. Sum of the areas of two squares is 400 cm^2. If the difference of their perimeters is 16 cm, find the sides of two squares.

Solution:

Area of a square $=s^2$
Perimeter of a square $=4$ s
Let the sides of the square be a and b respectively.
Given, sum of the areas of two squares is 400 cm^2 and the difference of their perimeters is 16 cm .
$\Rightarrow a^2+b^2=400$ and

$4a-4b=16$
$\Rightarrow a-b=4$
$\Rightarrow a=4+b$
$\Rightarrow (4+b)^2+b^2=400$
$\Rightarrow 2b^2+8b+16=400$
$\Rightarrow b^2+4b-192=0$
$\Rightarrow b^2+16b-12b-192=0$
$\Rightarrow (b+16)(b-12)=0$
$\Rightarrow b=12\ m$
Thus, $a=16\ m$.

Q9. The area of a rectangular plot is $528\ \mathrm{m}^2$. The length of the plot (in metres) is one metre more then twice its breadth. Find the length and the breadth of the plot.

Solution:

Given: The area of a rectangular plot is $528\ \mathrm{m}^2$. The length of the plot (in metres) is one metre more then twice its breadth.
To find: the length and the breadth of the plot.
Solution: Let the breadth be 'a' m
Given, length of the plot (in metres) is one metre more then twice its breadth.
$\Rightarrow$ length $=(2a+1)m$
Now, area of a rectangular plot is $528\ \mathrm{m}^2$
Area of rectangle $=$ length $\times$ breadth
$\Rightarrow a\times(2a+1)=528$
$\Rightarrow 2\mathrm{a}^2+\mathrm{a}-528=0$ In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.
$\Rightarrow 2a^2+33a-32a-528=0$
$\Rightarrow a(2a+33)-16(2a+33)=0$
$\Rightarrow (2a+33)(a-16)=0\Rightarrow(2a+33)=0$
and $(a-16)=0\Rightarrow 2a=-33$
and $a=16\Rightarrow a=-\frac{33}{2}$ and $a=16$
As any side can never be negative,
$\Rightarrow a=16$ m. length $=(2a+1)=2\times16+1=33$m
Hence breadth $=16$ m and length $=33$ m.

Exercise 8.12

Q1. A takes 10 days less than the time taken by B to finish a piece of work. If both A and B together can finish the work in 12 days, find the time taken by B to finish the work.

Solution:

Let the number of days in which B finishes the work be ' b '.
$\therefore$ Number of days in which A finishes the work $=b-10$

In 1 day,

B finishes $\frac{1}{b}$ of the work

A finishes $\frac{1}{b-10}$ of the work

Now, both A and B together can finish the work in 12 days

$\Rightarrow \frac{1}{b}+\frac{1}{b-10}=\frac{1}{12}$

$\Rightarrow 12(b-10+b)=b^2-10b$

$\Rightarrow 24\,b-120=b^2-10\,b$

$\Rightarrow b^2-34\,b+120=0$

$\Rightarrow b^2-30\,b-4\,b+120=0$

$\Rightarrow b(b-30)-4(\,b-30)=0$

$\Rightarrow b=4,30$ b can't be 4 as A takes 10 days less than B.

Thus number of days in which B alone finishes the work is 30 days.

Q2. If two pipes function simultaneously, a reservoir will be filled in 12 hours. One pipe fills the reservoir 10 hours faster than the other. How many hours will the second pipe take to fill the reservoir?

Solution:

Let the slower pipe fill the reservoir in $'a'$ hours

Faster pipe fills it in $'a\ -\ 10'$ hours.

Given, the two pipes will fill the reservoir together in 12 hours.

In 1 hour, part of reservoir filled $=\frac{1}{12}$

$\Rightarrow \frac{1}{a}+\frac{1}{a-10}=\frac{1}{12}$

$\Rightarrow 12(a+a-10)=a^2-10a$

$\Rightarrow 24a-120=a^2-10a$

$\Rightarrow \mathrm{a}^2-34\mathrm{a}+120=0$

$\Rightarrow a^2-30a-4a+120=0$

$\Rightarrow a(a-30)-4(a-30)=0$

$\Rightarrow (a-4)(a-30)=0$

Value of a can't be 4 as $(a-10)$ will be negative

Thus $a=30$.

Q3. Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

Solution:

Let the smaller diameter tap fill the reservoir in ' a ' hours

Larger diameter tap fills it in 'a - 10' hours.

Given, two water taps together can fill a tank in $9\frac{3}{8}=\frac{75}{8}$ hours.

In 1 hour, part of tank filled $= \frac{8}{75}$

$\Rightarrow \frac{1}{a} + \frac{1}{a-10} = \frac{8}{75}$

$\Rightarrow 75(a + a - 10) = 8a^2 - 80a$

$\Rightarrow 150\text{a} - 750 = 8\text{a}^2 - 80\text{a}$

$\Rightarrow 8a^2 - 230a + 750 = 0$

$\Rightarrow 4a^2 - 115a + 375 = 0$

$\Rightarrow 4\text{a}^2 - 100\text{a} - 15\text{a} + 375 = 0$

$\Rightarrow 4a(a - 25) - 15(a - 25) = 0$

$\Rightarrow (4a - 15)(a - 25) = 0$

Value of a can't be $\frac{15}{4}$ as $(a - 10)$ will be negative

Thus $a = 25$

Time taken by faster tap $= 25 - 10 = 15$ hours.

Q4. Two pipes running together can fill a tank in $11\frac{1}{9}$ minutes. If one pipe takes 5 minutes more than the other to fill the tank separately, find the time in which each pipe would fill the tank separately.

Solution:

Let the faster pipe fill the tank in ' a ' min

Slower pipe fills it in ' $a + 5$ ' min.

Given, the pipes running together can fill a tank in $11\frac{1}{9} = \frac{100}{9}$ minutes.

In 1 min, part of tank filled $= \frac{9}{100}$

$\Rightarrow \frac{1}{a} + \frac{1}{a+5} = \frac{9}{100}$

$\Rightarrow 100(a + a + 5) = 9(a^2 + 5a)$

$\Rightarrow 200a + 500 = 9a^2 + 45a$

$\Rightarrow 9a^2 - 155a - 500 = 0$

$\Rightarrow 9a^2 - 180a + 25a - 500 = 0$

$\Rightarrow 9a(a - 20) + 25(a - 20) = 0$

$\Rightarrow (9a + 25)(a - 20) = 0$

$\Rightarrow a = 20$ mins

Slower pipe will fill it in 25 min.

Q5. To fill a swimming pool two pipes are used. If the pipe of larger diameter used for 4 hours and the pipe of smaller diameter for 9 hours, only half of the pool can be filled. Find, how long it would take for each pipe to fill the pool separately, if the pipe of smaller diameter takes 10 hours more than the pipe of larger diameter to fill the pool?

Solution:

Let the larger diameter pipe fill it in $'a'$ hours

The smaller diameter pipe fills it in $'a + 10'$ hours
In 1 hour, larger diameter pipe fills $1/a$ part of the pool.
In 1 hour, smaller diameter pipe fills $\frac{1}{a+10}$ part of the pool.
Given, the pipe of larger diameter used for 4 hours and the pipe of smaller diameter for 9 hours, only half of the pool can be filled.

$$\Rightarrow 4 \times \frac{1}{a} + 9 \times \frac{1}{a+10} = \frac{1}{2}$$
$$\Rightarrow 2(4a + 40 + 9a) = a^2 + 10a$$
$$\Rightarrow 26a + 80 = a^2 + 10a$$
$$\Rightarrow a^2 - 16a - 80 = 0$$
$$\Rightarrow a^2 - 20a + 4a - 80 = 0$$
$$\Rightarrow a(a - 20) + 4(a - 20) = 0$$
$$\Rightarrow (a + 4)(a - 20) = 0$$
$$\Rightarrow a = 20\ hours$$

Time in which smaller diameter pipe fills the pool $= 20 + 10 = 30$ hours.

Exercise 8.13

Q1. A piece of cloth costs ₹35. If the piece were 4 m longer and each metre costs ₹1 less, the cost would remain unchanged. How long is the piece?

Solution:

Let the length of cloth be ' a ' m.
Given, piece of cloth costs ₹35 and if the piece were 4 m longer and each metre costs ₹1 less, the cost remains unchanged.
Cost of 1 m of cloth $= \frac{35}{\mathrm{a}}$

$$\Rightarrow (a + 4) \times \left(\frac{35}{a} - 1\right) = 35$$
$$\Rightarrow (a + 4)(35 - a) = 35a$$
$$\Rightarrow 35a + 140 - a^2 - 4a = 35a$$
$$\Rightarrow a^2 + 4a - 140 = 0$$
$$\Rightarrow a^2 + 14a - 10a - 140 = 0$$
$$\Rightarrow a(a + 14) - 10(a + 14) = 0$$
$$\Rightarrow (a - 10)(a + 14) = 0$$
$$\Rightarrow a = 10\ m$$

Q2. Some students planned a picnic. The budget for food was ₹480. But eight of these failed to go and thus the cost of food for each member increased by ₹10. How many students attended the picnic?

Solution:

Let the number of students who planned the picnic be ' a '.

Budget for the food was ₹ 480

Cost of food for each member $= \frac{480}{a}$

Given, eight of these failed to go and thus the cost of food for each member increased by ₹ 10

$\Rightarrow (a-8) \times \left(\frac{480}{a} + 10\right) = 480$

$\Rightarrow (a-8)(480+10a) = 480a$

$\Rightarrow 480a + 10a^2 - 3840 - 80a = 480a$

$\Rightarrow a^2 - 8a - 384 = 0$

$\Rightarrow a^2 - 24a + 16a - 384 = 0$

$\Rightarrow a(a-24) + 16(a-24) = 0$

$\Rightarrow (a+16)(a-24) = 0$

$\Rightarrow a = 24$

Number of students who attended the picnic $= 24 - 8 = 16$.

Q3. A dealer sells an article for ₹24 and gains as much percent as the cost price of the article. Find the cost price of the article.

Solution:

Let the cost price be ₹ a.

Given, the dealer sells an article for ₹ 24 and gains as much percent as the cost price of the article.

It's given that he gains as much as the cost price of the article, thus, Gain % = a%

Gain % $= \frac{SP-CP}{CP} \times 100$

$\Rightarrow a = \frac{24-a}{a} \times 100$

$\Rightarrow a^2 = (24-a) \times 100$

$\Rightarrow a^2 + 100a - 2400 = 0$

$\Rightarrow a^2 + 120a - 20a - 2400 = 0$

$\Rightarrow (a+120)(a-20) = 0$

$\Rightarrow a = 20$ or -120

Since money cannot be negative so, negalecting -120, we get,

$\Rightarrow a = 20$

Thus, the cost price of the article is ₹20.

Q4. Out of a group of swans, $\frac{7}{2}$ times the square root of the total number are playing on the share of a pond. The two remaining ones are swinging in water. Find the total number of swans.

Solution:

Let the number of swans in the pond be $'a'$.

Given, out of a group of swans, $\frac{7}{2}$ times the square root of the total number are playing on the share of a pond. The two remaining ones are swinging in water.

$\Rightarrow \frac{7}{2}\sqrt{a}+2=a$

$\Rightarrow 7\sqrt{a}=2a-4$

Squaring both sides

$\Rightarrow 49a=4a^2+16-16a$

$\Rightarrow 4a^2-65a+16=0$

$\Rightarrow 4a^2-64a-a+16=0$

$\Rightarrow 4a(a-16)-(a-16)=0$

$\Rightarrow (4a-1)(a-16)=0$

$\Rightarrow a$ can't be $\frac{1}{4}$, thus $a=16$.

Q5. If the list price of a toy is reduced by ₹2, a person can buy 2 toys more for ₹ 360. Find the original price of the toy.

Solution:

Let the original price of the toy be ' a '.

Given, when the list price of a toy is reduced by ₹ 2, a person can buy 2 toys more for ₹360.

The number of toys he can buy at the original price for ₹ 360 $=\frac{360}{\mathrm{a}}$

According to the question,

$\Rightarrow \frac{360}{a-2}=\frac{360}{a}+2$

$\Rightarrow 360a=(a-2)(360+2a)$

$\Rightarrow 360a=360a+2a^2-720-4a$

$\Rightarrow a^2-2a-360=0$

$\Rightarrow a^2-20a+18a-360=0$

$\Rightarrow a(a-20)+18(a-20)=0$

$\Rightarrow (a+18)(a-20)=0 \Rightarrow a+18=0$ or $a-20=0 \Rightarrow a=-18$ or $a=20$

As, price can't be negative, $a=-18$ is not possible

Therefore, $a=$ ₹20.

Q6. ₹ 9000 were divided equally among a certain number of persons. Had there been 20 more persons, each would have got ₹ 160 less. Find the original number of persons.

Solution:

Let the original number of people be ' a '.

Given, ₹9000 were divided equally among a certain number of persons. Had there been 20 more persons, each would have got ₹ 160 less.

Amount which each receives $=\frac{9000}{a}$

$\Rightarrow \frac{9000}{a+20}=\frac{9000}{a}-160$

$\Rightarrow 9000a=(9000-160a)(a+20)$

$\Rightarrow 9000a=9000a+180000-160a^2-3200a$

$\Rightarrow a^2 + 20a - 1125 = 0$
$\Rightarrow a^2 + 45a - 25a - 1125 = 0$
$\Rightarrow a(a + 45) - 25(a + 45) = 0$
$\Rightarrow (a - 25)(a + 45) = 0$
$\Rightarrow a = 25.$

Q7. Some students planned a picnic. The budget for food was ₹500. But, 5 of them failed to go and thus the cost of food for each member increased by ₹ 5. How many students attended the picnic?

Solution:

Let the number of students who planned the picnic be ' a '.
Budget for the food was ₹500
Cost of food for each member $= \frac{500}{a}$
Given, 5 of these failed to go and thus the cost of food for each member increased by ₹ 5

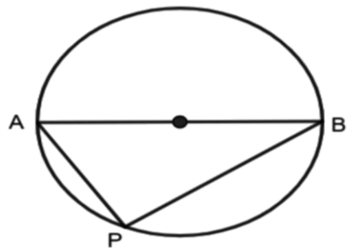
$\Rightarrow (a - 5) \times \left(\frac{500}{a} + 5\right) = 500$
$\Rightarrow (a - 5)(100 + a) = 100a$
$\Rightarrow 100a + a^2 - 500 - 5a = 100a$
$\Rightarrow a^2 - 5a - 500 = 0$
$\Rightarrow a^2 - 25a + 20a - 500 = 0$
$\Rightarrow a(a - 25) + 20(a - 25) = 0$
$\Rightarrow (a + 20)(a - 25) = 0$
$\Rightarrow a = 25$

Number of students who attended the picnic $= 25 - 5 = 20$.

Q8. A pole has to be erected at a point on the boundary of a circular park of diameter 13 metres in such a way that the difference of its distances from two diametrically opposite fixed gates A and B on the boundary is 7 metres. Is it the possible to do so? If yes, at what distances from the two gates should the pole be erected?

Solution:

Let the distance of pole from gate A be ' a '.
$\Rightarrow$ Difference of the distance of the pole from two diametrically opposite fixed gates A and B on the boundary is 7 metres.
$\Rightarrow$ Distance of pole from gate $B = a - 7m$
$\Rightarrow$ Diameter of the park $= 13$ m



Now, Angle in a semicircle is a right angle, therefore ABP is a triangle right-angled at P, Therefore, By Pythagoras theorem i.e., $Hypotenuse^2 = length^2 + breadth^2$

$\Rightarrow 13^2 = a^2 + (a-7)^2$

$\Rightarrow 169 = 2a^2 + 49 - 14a$

$\Rightarrow a^2 - 7a - 60 = 0$

$\Rightarrow a^2 - 12a + 5a - 60 = 0$

$\Rightarrow a(a-12) + 5(a-12) = 0$

$\Rightarrow (a+5)(a-12) = 0$

$\Rightarrow a = -5 \; or \, a = 12 \; m$

but distance can't be negative, hence $a = 12$

Thus, distance of pole is 12 m from gate A and $(12-7) = 5$ meters from gate B.

Q9. In a class test, the sum of the marks obtained by P in Mathematics and science is 28 . Had he got 3 marks more in Mathematics and 4 marks less in Science. The product of his marks, would have been 180. Find his marks in the two subjects.

Solution:

Given: In a class test, the sum of the marks obtained by P in Mathematics and science is 28. Had he got 3 marks more in Mathematics and 4 marks less in Science. The product of his marks, would have been 180.

To find: his marks in the two subjects.

Let the marks obtained in Mathematics by P be $'a'$.

Given, sum of the marks obtained by P in Mathematics and science is 28.

$\Rightarrow$ Marks obtained in science $= 28 - a$

Also, if he got 3 marks more in Mathematics and 4 marks less in Science, product of his marks, would have been 180.

$\Rightarrow (a+3)(28-a-4) = 180 \Rightarrow (a+3)(24-a) = 180$

$\Rightarrow 24a - a^2 + 72 - 21a = 180$

$\Rightarrow -a^2 + 21a + 72 = 180 \Rightarrow -a^2 + 21a + 72 - 180 = 0 \Rightarrow -a^2 + 21a - 108 = 0$

$\Rightarrow a^2 - 21a + 108 = 0$

Factorise the above quadratic equation by splitting the middle term:

$\Rightarrow a^2 - 12a - 9a + 108 = 0$

$\Rightarrow a(a-12) - 9(a-12) = 0$

$\Rightarrow (a-9)(a-12) = 0$

$\Rightarrow a = 9$ or 12If marks obtained in mathematics is 9 , the marks obtained in science is $28 - a = 28 - 9 = 19$

$\Rightarrow$ Marks in Mathematics $= 9$, Marks in Science $= 19$

If marks obtained in mathematics is 12 , the marks obtained in science is

$28 - \mathrm{a} = 28 - 12 = 16$

Marks in Mathematics $= 12$, Marks in Science $= 16$.

Q10. In a class test, the sum of Shefali's marks in Mathematics and English is 30. Had she got 2 marks more in Mathematics and 3 marks less in English, the product of her marks would have been 210. Find her marks in two subjects.

Solution:

Given: In a class test, the sum of Shefali's marks in Mathematics and English is 30 . Had she got 2 marks more in Mathematics and 3 marks less in English, the product of her marks would have been 210.

To find: her marks in two subjects.

Let the marks obtained in Mathematics by Shefali be ' a '.

Given, sum of the marks obtained by Shefali in Mathematics and English is 30.

Marks obtained in english $= 30 - \text{a}$

Also, she got 2 marks more in Mathematics and 3 marks less in English, the product of her marks would have been 210.

$\Rightarrow (a+2)(30-a-3)=210 \Rightarrow (a+2)(27-a)=210$

$\Rightarrow 27a - a^2 + 54 - 2a = 210$

$\Rightarrow -a^2 + 25a + 54 = 210 \Rightarrow -a^2 + 25a + 54 - 210 = 0 \Rightarrow -a^2 + 25a - 156 = 0$

$\Rightarrow a^2 - 25a + 156 = 0$

In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

$\Rightarrow a^2 - 13a - 12a + 156 = 0$

$\Rightarrow a(a-13) - 12(a-13) = 0$

$\Rightarrow (a-12)(a-13) = 0$

$\Rightarrow a = 12 \textit{ or } 13$

If marks in mathematics is 12 marks in English is $30 - a = 30 - 12 = 18$ If marks in mathematics is 13 marks in English is $30 - a = 30 - 13 = 17$

Hence

Marks in Mathematics $= 12$, Marks in English $= 18$

Marks in Mathematics $= 13$, Marks in English $= 17$.

Q11. A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If the total cost of production on that day was ₹90, find the number of articles produced and the cost of each article.

Solution:

Let the number of article produced on the day be ' a '.

Given, it was observed on a particular day that the cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day.

Cost of production of each article $= 2a + 3$

Given, cost of production was Rs. 90

$\Rightarrow a(2a+3) = 90$

$\Rightarrow 2a^2 + 3a - 90 = 0$
$\Rightarrow 2a - 12a + 15a - 90 = 0$
$\Rightarrow 2a(a-6) + 15(a-6) = 0$
$\Rightarrow (2a+15)(a-6) = 0$
$\Rightarrow a = 6$
Cost of each article $= 2 \times 6 + 3 =$ ₹15.

CCE - Formative Assessment

Q1. Write the value of k for which the quadratic equation $x^2 - kx + 4 = 0$ has equal roots.

Solution:

Quadratic equation has equal roots then $d = b^2 - 4ac = 0$
Here $a = 1, b = k$ and $c = 4$
So $b^2 - 4ac = 0$
$\Rightarrow k^2 - 4 \times 1 \times 4 = 0$
$\Rightarrow k^2 - 16 = 0$
$\Rightarrow k = \pm 4$.

Q2. What is the nature of roots of the quadratic equation $4x^2 - 12x - 9 = 0$?

Solution:

Consider the equation $4x^2 - 12x - 9 = 0$,To check the roots of the equation we will check the value of $d = b^2 - 4ac$
Here $a = 4, b = 12$ and $c = -9$
So $d = (12)^2 - 4 \times 4 \times -9$
$= 144 + 144$
$= 288 > 0$ which is real
So the roots of the given equation are Real and distinct.

Q3. If $1 + \sqrt{2}$ is a root of a quadratic equation with rational coefficients, write its other root.

Solution:

$1 + \sqrt{2}$ Is a root of quadratic equation with rational coefficients that is the sum of the roots is rational and the product of the roots is also rational.
Since the rational roots occurs in conjugate pairs so the other root of the equation is $1 - \sqrt{2}$.

Q4. Write the number of real roots of the equation $x^2 + 3|x| + 2 = 0$.

Solution:

If $ax^2+bx+c=0$ then $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$
If $D=b^2-4ac\geq 0$ then the values of x are real
If $D=b^2-4ac<0$ then the values of x are complex
$|z|$ is always a positive real number regardless of x being a real number or complex number.
Given eqn. is $|x|^2+3|x|+2=0$ and $a=1, b=3$ and $c=2$
$|x|^2+3|x|+2=0$
$\Rightarrow |x|=\frac{-3\pm\sqrt{9-8}}{2}$
$\Rightarrow |x|=-2$ or -1
But $|x|$ cannot be negative
No real root for the equation.

Q5. Write the sum of real roots of the equation $x^2+|x|-6=0$.

Solution:

First of all, the equation is $x^2+|x|-6=0$
CASE 1: $x>0$ then $|x|=x$
$\Rightarrow x^2+x-6=0$
$\Rightarrow x^2+3x-2x-6=0$
$\Rightarrow (x+3)(x-2)=0$
$\Rightarrow x=2,-3$
CASE 2: $\mathrm{x}<0|\mathrm{x}|=-\mathrm{x}$
$\Rightarrow x^2-x-6=0$
$\Rightarrow x^2-3x+2x-6=0$
$\Rightarrow (x-3)(x+2)=0$
$\Rightarrow x=-2,3$
So the sum of the roots is $2+(-3)+(-2)+3$
$=0$

Q6. Write the set of values of ' a ' for which the equation $x^2+ax-1=0$ has real roots.

Solution:

Consider $x^2+ax-1=0$,For the quadratic equation to have real roots $D\geq 0$
Here $a=1, b=a$ and $c=1$
In the given equation $D=a^2-4\geq 0$
$\Rightarrow a^2\geq 4$
So for all the real values of 'a' which are greater than or equal to 2 and -2 the equation will have the real roots.

Q7. Is there any real value of 'a' for which the equation $x^2+2x+(a^2+1)=0$ has real roots?

Solution:

A quadratic equation has two real roots if discriminant $= 0$
For the given equation, we have:
$d = b^2 - 4ac$
$d = (2)^2 - 4(1)(a^2 + 1)$
$d = 4 - 4(a^2 + 1)$
$d = 4(1 - a^2 - 1)$
$d = -4a^2$
Now, $D = 0$ when $a = 0$. So, the equation will have real and equal roots if $a = 0$. And for all other values of a, the equation will have no real roots.
No, there is no real value of ' a ' for which the given equation has real roots.

Q8. Write the value of λ for which $x^2 + 4x + \lambda$, is a perfect square.
Solution:
For being the perfect square, the roots are equal
So, $d = b^2 - 4ac = 0$
Here $a = 1, b = 4$ and $c = \lambda$
$\Rightarrow d = 16 - 4\lambda = 0$
$\Rightarrow \lambda = 4$

Q9. Write the condition to be satisfied for which equations $ax^2 + 2bx + c = 0$ and $bx^2 - 2\sqrt{ac}x + b = 0$ have equal roots.
Solution:
Given the roots of both the equations are real
For first equation $ax^2 + 2bx + c = 0$
Its discriminant; $d \geq 0$
$D = b^2 - 4ac$
$D = (2b)^2 - 4 \times a \times c \geq 0$
$4b^2 \geq 4ac$
$b^2 \geq ac \ldots 1$
For second equation
$bx^2 - 2\sqrt{ac}x + b = 0$
$d = b^2 - 4ac \geq 0$
$= (2\sqrt{ac})^2 - 4bxb \geq 0$
$= 4ac - 4b^2 \geq 0$
$ac \geq b^2 \ldots 2$
From 1 and 2 we get only one case where $b^2 = ac$.

Q10. Write the set of values of k for which the quadratic equation has $2x^2 + kx + 8 = 0$ has real roots.
Solution:

To have the real roots $D = b^2 - 4ac \geq 0$
Here $a = 2, b = k$ and $c = 8$
$D = k^2 - 4 \times 2 \times 8 \geq 0$
$\Rightarrow K^2 \geq 64$
$\Rightarrow K \geq \pm 8$
So for all the values of k greater than or equal to 8 and -8 , the given quadratic equation will have real roots.

Q11. Write a quadratic polynomial, sum of whose zeros is $2\sqrt{3}$ and their product is 2.
Solution:
The sum of the two zeros of the quadratic equation is given by $-\frac{b}{a}$
Here it's given $-\frac{b}{a} = 2\sqrt{3}$
The product of the quadratic equation is $\frac{c}{a}$
Here $\frac{c}{a} = 2$
the quadratic equation is of the form $ax^2 + bx + c = 0$
or $x^2 +$ (sum of the roots) $x +$ product of the roots $= 0$
$= x^2 - 2\sqrt{3}x + 2$
$f(x) = k\left(x^2 - 2\sqrt{3}x + 2\right)$, where k is any real number.

Q12. Show that $x = -3$ is a solution of $x^2 + 6x + 9 = 0$.
Solution:
To be the solution of the equation the value $x = -3$ should satisfy the given equation
$x^2 + 6x + 9 = 0$ Putting value of x on L.H.S
$(-3)^2 + 6 \times (-3) + 9$
$\Rightarrow 9 - 18 + 9 = 0 =$ R.H.S
Hence $x = -3$ is the solution of given equation.

Q13. Show that $x = -2$ is a solution of $3x^2 + 13x + 14 = 0$.
Solution:
To be solution of the equation $x = -2$ should satisfy the given equation
L.H.S $3x(-2)^2 + 13x - 2 + 14$
$\Rightarrow 12 - 26 + 14 = 0 =$ R.H.S
Hence $x = -2$ is the solution of the equation.

Q14. Find the discriminant of the quadratic equation $3\sqrt{3x^2} + 10x + \sqrt{3} = 0$
Solution:
$d = b^2 - 4ac$
Here $a = 3\sqrt{3}$
$B = 10$ and $c = \sqrt{3}$
$D = (10)^2 - 4(3\sqrt{3})\sqrt{3}$

$D = 100 - 36$
$D = 64.$

Q15. If $x = \frac{-1}{2}$, is a solution of the quadratic equation $3x^2 + 2kx - 3 = 0$, find the value of k.

Solution:

Since $x = \frac{-1}{2}$ is the solution of the equation it should satisfy the equation

Putting value of x in the given equation

$3\left(-\frac{1}{2}\right)^2 + 2k\left(-\frac{1}{2}\right) - 3 = 0$

$\frac{3}{4} - k - 3 = 0$

$\frac{9}{4} = k$

Q1. If the equation $x^2 + 4x + k = 0$ has real and distinct roots, then

A. $k < 4$
B. $k > 4$
C. $k \geq 4$
D. $k \leq 4$

Solution:

If roots of given equation are real and distinct then $D = b^2 - 4ac > 0$

Here $a = 1, b = 4$ and $c = k$

So, $4^2 - 4(1)(k) > 0$

$16 - 4k > 0$

$16 > 4k$

$K < 4.$

Q2. If the equation $x^2 - ax + 1 = 0$ has two distinct roots, then

A. $|a| = 2$
B. $|a| < 2$
C. $|a| > 2$
D. None of these

Solution:

If roots of given equation are distinct then

$d = b^2 - 4ac > 0$

Here $a = 1, b = a, c = 1$

So, $a^2 - 4(1)(1) > 0$

$a^2 - 4 > 0$

$a^2 > 4$

$|a| > 2.$

Q3. If the equation $9x^2 + 6kx + 4 = o$ has equal roots, then the roots are both equal to?

A. $\pm\frac{3}{2}$

B. $\pm\frac{3}{2}$

C. 0

D. ± 3

Solution:

Given: the equation $9x^2 + 6kx + 4 = 0$ has equal roots.

To find: the roots are both equal to ?

If roots of given equation are equal then $D = b^2 - 4ac = 0$

$\Rightarrow (6k)^2 - 4(9)(4) = 0$

$\Rightarrow 36k^2 - 144 = 0$

$\Rightarrow 36k^2 = 144$

$\Rightarrow K^2 = 4$

$\Rightarrow K = \pm 2$

Case 1 :- when $k = 2$

In equation $9x^2 + 6kx + 4 = 0$

$9x^2 + 6(2)x + 4 = 0$

$9x^2 + 12x + 4 = 0$

$(3x)^2 + 2 \times 2 \times 3x + (2)^2 = 0$

$(3x + 2)^2 = 0$

$3x + 2 = 0$

$3x = -2$

$$x = -\frac{2}{3}$$

Case 2 :- when $k = -2$

In equation $9x^2 + 6kx + 4 = 0$

$9x^2 + 6(-2)x + 4 = 0$

$9x^2 - 12x + 4 = 0$

$(3x)^2 - 2 \times 2 \times 3x + (2)^2 = 0$

$(3x - 2)^2 = 0$

$3x - 2 = 0$

$3x = 2$

$x = \frac{2}{3}$ So the roots of the given quadratic equation are $\pm\frac{2}{3}$.

Q4. If $ax^2 + bx + c = 0$ has equal roots, then $c =$

A. $-b/2a$

B. $\frac{b}{2a}$

C. $\frac{-b^2}{4a}$

D. $\frac{b^2}{4a}$

Solution:

Let the roots of given equation be m and n

According to the question $M = n$

Sum of roots $= m + n = -\frac{b}{a}$

$2\,m = -\dfrac{b}{a}$

$\Rightarrow m = -\dfrac{b}{2a}$

Product of roots $= m \times n = \frac{c}{a}$

$m^2 = \dfrac{c}{a}$

$(-b/2a)^2 = \dfrac{c}{a}$

$\dfrac{b^2}{4a^2} = \dfrac{c}{a}$

$\dfrac{b^2}{4a^2} = c$

Therefore $c = \frac{b^2}{4a}$.

Q5. If the equation $ax^2 + 2x + a = 0$ has two distinct roots, if

A. $a = \pm 1$

B. $a = 0$

C. $a = 0,1$

D. $a = -1,0$

Solution:

If the roots of given equation are distinct then

$d = b^2 - 4ac = 0$

$\Rightarrow d = b^2 - 4ac = 0$

$\Rightarrow 2^2 - 4(a)(a) = 0$

$\Rightarrow 4 - 4a^2 = 0$

$\Rightarrow 4a^2 = 4$

$\Rightarrow a^2 = 1$

$\Rightarrow a = \pm 1.$

Q6. The positive value of k for which the equation $x^2 + kx + 64 = 0$ and $x^2 - 8x + k = 0$ will both have real roots, is

A. 4

B. 8

C. 12

D. 16

Solution:

If the given equation $x^2 + kx + 64$ has real roots then $D \geq 0$

$D = b^2 - 4ac \geq 0$

Here $a = 1, b = k, c = 64$

$d = K^2 - 4(1)(64) \geq 0$
$d = K^2 - 256 \geq 0$
$K \geq 16$......
If the given equation $x^2 - 8x + k = 0$ has real roots then then
$d \geq 0$
$D = b^2 - 4ac \geq 0$
$8^2 - 4(1)(k) \geq 0$
$64 - 4k \ \geq 0$
$64 \geq 4\text{k}$
$K \leq 16$......
From (1) and (2) we can conclude that $k = 16$.

Q7. The value of $\sqrt{6 + \sqrt{6 + \sqrt{6 + \cdots}}}$ is
A. 4
B. 3
C. -2
D. 3.5
Solution:
In given equation let $x = \sqrt{6 + \sqrt{6 + \sqrt{6 + \cdots}}}$
So, $x = \sqrt{(6 + x)}$
Now squaring both side
$x^2 = 6 + x$
$x^2 - x - 6 = 0$
$x^2 - 3x + 2x - 6 = 0$
$x(x - 3) + 2(x - 3) = 0$
$(x - 3)(x + 2) = 0$
$x = 3$ or -2
x cannot be equal to -2 as root can never be negative.
$x = 3$.

Q8. If 2 is a root of the equation $x^2 + bx + 12 = 0$ and the equation $x^2 + bx + q = 0$ has equal roots, then $q =$
A. 8
B. -8
C. 16
D. -16
Solution:
2 is the root of given equation $x^2 + bx + 12 = 0$
So $2^2 + 2b + 12 = 0$
$16 + 2b = 0$

$b=-8 \quad .1$

Now, $d=b^2-4ac=0$ of second equation is

$d=b^2-4(1)(q)=0$ here $a=1, b=-8($ from 1) and $c=q$

$(-8)^2-4q=0$

$64-4q=0$

$q=16$

Hence value of q is 16 .

Q9. If the equation $(a^2+b^2)x^2-2(ac+bd)x+c^2+d^2=0$ has equal roots, then

A. $ab=cd$

B. $ad=bc$

C. $\mathrm{ad}=\sqrt{\mathrm{bc}}$

D. $ab=\sqrt{cd}$

Solution:

If the roots are equal then $d=b^2-4ac=0$.

Here $a=(a^2+b^2), b=2(ac+bd), c=(c^2+d^2)$

$D=b^2-4ac=0$

$\Rightarrow b^2=4\mathrm{ac}$

$\Rightarrow \{-2(ac+bd)\}^2=4\{(a^2+b^2)(c^2+d^2)\}$

$\Rightarrow 4(a^2c^2+b^2d^2+2acbd)=4(a^2c^2+a^2d^2+b^2c^2+b^2d^2)$

$\Rightarrow 2acbd=a^2d^2+b^2c^2$

$\Rightarrow a^2d^2+b^2c^2-2abcd=0$

$\Rightarrow (ad-bc)^2=0$

$\Rightarrow ad-bc=0$

$\Rightarrow ad=bc$

Q10. If the roots of the equation $(a^2b^2)x^2-2b(a+c)x+(b^2+c^2)=0$ are equal, then

A. $2b=a+c$

B. $b^2=ac$

C. $\mathrm{b}=\frac{2\mathrm{ac}}{\mathrm{a+c}}$

D. $b=ac$

Solution:

The roots of the equation are equal so $d=b^2-4ac=0$

Here $a=(a^2+b^2), b=-2\,b(a+c), c=(b^2+c^2)$

$d=(-2b(a+c))^2=4(a^2+b^2)(b^2+c^2)$

$\Rightarrow b^2(a^2+2ac+c^2)=a^2b^2+a^2c^2+b^4+b^2c^2$

$\Rightarrow (\mathrm{ac})^2-2(\mathrm{ac})(\mathrm{b}^2)+(\mathrm{b}^2)^2=0$

$\Rightarrow (\mathrm{ac}-\mathrm{b}^2)^2=0$

$\Rightarrow (\mathrm{ac}-\mathrm{b}^2)=0$

$\Rightarrow ac=b^2$.

Q11. If the equation $x^2 - bx + 1 = 0$ does not possess real roots, then

A. $-3 < b < 3$

B. $-2 < b < 2$

C. $b > 2$

D. $b < -2$

Solution:

If the equation does not possess real roots then

$d = b^2 - 4ac$

Here $a = 1, b = -b, c = 1$

$d = b^2 - 4 < 0$

$b^2 < 4$

$b < \pm 2$

$-2 < b < 2.$

Q12. If $x = 1$ is a common root of the equations $ax^2 + ax + 3 = 0$ and $x^2 + x + b = 0$, then $ab =$

A. 3

B. 3.5

C. 6

D. -3

Solution:

Since $x = 1$ is root of equations, it will satisfy both the equations.

Putting $x = 1$ in $ax^2 + ax + 3 = 0$

$1a + a + 3 = 0$

$2a + 3 = 0$

$A = -\frac{3}{2}$

Putting $x = 1$ in $x^2 + x + b = 0$

$1 + 1 + b = 0$

$b = -2$

$ab = \frac{-3}{2} x - 2$

$ab = 3.$

Q13. If p and q are the roots of the equation $x^2 + px + q = 0$, then

A. $p = 1, q = -2$

B. $q = 0, p = 1$

C. $p = -2, q = 0$

D. $p = -2, q = 1$

Solution:

Since p and q are roots of the equations then

Sum of the roots is $p + q = -\frac{b}{a} = -(p) = -p$

Here $a = 1, b = p$ and $c = q$

Products of the root $= p \times q = \frac{c}{a} = q$

$\therefore p \times q = q$

$p = 1$

Putting value of $'p'$ in $p + q = -p$

$1 + q = -1$

$q = -2.$

Q14. If a and b can take values 1,2,3,4. Then the number of the equations of the form $ax^2 + bx + 1 = 0$ having real roots is

A. 10

B. 7

C. 6

D. 12

Solution:

For quadratic equation to have real roots,

$d \geq 0$

$b^2 - 4a \geq 0$

$b^2 \geq 4a$

For $a = 1, 4a = 4, b = 2,3,4$ (3 equations)

With values of (a, b) as $(1,2), (1,3), (1,4)$

$a = 2, 4a = 8, b = 3,4$ (2 equations)

With values of a, b as $(2,3), (2,4)$

$a = 3, 4a = 12, b = 4$ (1 equation)

With value of (a, b) as $(3,4)$

$a = 4, 4a = 16, b = 4$ (1 equation)

With values of (a, b) as $(4,16)$

Thus, total 7 equations are possible.

Q15. The number of quadratic equations having real roots and which do not change by squaring their roots is

A. 4

B. 3

C. 2

D. 1

Solution:

The roots of the equation are real (given)

Let a and β be the two roots according to the given condition

$a = a^2$

$\beta = \beta^2$

Sum of the roots $= a + \beta = a^2 + \beta^2$

Product of the roots $= \alpha\beta = \alpha^2\beta^2$
There are only two number who does not change on squaring them that is 0 and 1
So the number of equations could be 2 by being the roots as
(0,1) and (1,0).

Q16. If $(a^2 + b^2)x^2 + 2(ac + bd)x + c^2 + d^2 = 0$ has no real roots, then

A. ad = bc
B. $ab = cd$
C. ac = bd
D. $ad \neq bc$

Solution:

Since the equation
$(a^2 + b^2)x^2 + 2(ac + bd)x + c^2 + d^2 = 0$ has no real root
D < 0
$b^2 - 4ac < 0$
$b^2 < 4ac$
Here $a = (a^2 + b^2), b = 2(ab + bd), c = c^2 + d^2$
$4(ac + bd)^2 - 4(a^2 + b^2)(c^2 + d^2) < 0$
$4a^2c^2 + 4b^2d^2 + 8abcd - 4(a^2c^2 + b^2c^2 + a^2d^2 + b^2d^2) < 0$
$-4(a^2d^2 + b^2c^2 - 2abcd) < 0$
$-4(ad + bc)^2 < 0$
$\therefore d$ is always negative
And $ad \neq bc$.

Q17. If the sum of the roots of the equation $x^2 - x = \lambda(2x - 1)$ is zero, then $\lambda =$

A. -2
B. 2
C. $-\frac{1}{2}$
D. $\frac{1}{2}$

Solution:

equation is $x^2 - x = \lambda(2x - 1)$
$x^2 - x - \lambda(2x - 1) = 0$
$x^2 - (2\lambda + 1)x + \lambda = 0$
Here $a = 1, b = -(2\lambda + 1), c = \lambda$
Sum of the roots $= -\frac{b}{a}$
$\Rightarrow -(-(2\lambda + 1)) = 0$
$\Rightarrow \lambda = -\frac{1}{2}$.

Q18. If $x = 1$ is a common root of $ax^2 + ax + 2 = 0$ and $x^2 + x + b = 0$ then, $ab =$

A. 1

B. 2
C. 4
D. 3

Solution:

Since $x = 1$ is root of equation
Then it satisfy the equation
Putting $x = 1$ in first equation
$a + a + 2 = 0$
$2a + 2 = 0$
$a = -1$
Putting $x = 1$ in equation second
$1 + 1 + b = 0$
$2 + b = 0$
$b = -2$
$ab = -1x - 2$
$ab = 2.$

Q19. The value of c for which the equation $ax^2 + 2bx + c = 0$ has equal roots is

A. $\frac{a^2}{4b}$
B. $\frac{b^2}{4a}$
C. $\frac{a^2}{b}$
D. $\frac{a^2}{4b}$

The equation has equal root which means $d = 0$
$d = b^2 - 4ac = 0$
Here $a = a, b = 2b, c = c$
$(2b)^2 - 4ac = 0$
$b^2 - ac = 0$
$c = \frac{b^2}{a}.$

Q20. If $x^2 + k(4x + k - 1) + 2 = 0$ has equal roots, then $k =$

A. $-\frac{2}{3}, 1$
B. $\frac{2}{3}, -1$
C. $\frac{3}{2}, \frac{1}{3}$
D. $-\frac{3}{2}, -\frac{1}{3}$

Solution:

Equation $x^2 + k(4x + k - 1) + 2 = 0$ has equal roots
$d = 0$

$d = b^2 - 4ac = 0$
Here $a = 1, b = 4k, c = k^2 - k + 2$
$\Rightarrow 16k^2 - 4(k^2 - k + 2) = 0$
$\Rightarrow 12k^2 + 4k - 8 = 0$
$\Rightarrow 3k^2 + k - 2 = 0$
$\Rightarrow (3k - 2)(k + 1) = 0$
$\Rightarrow k = \frac{2}{3}, -1.$

Q21. If the sum and product of the roots of the equation $kx^2 + 6x + 4k = 0$ are equal, then $k =$

A. $-\frac{3}{2}$
B. $\frac{3}{2}$
C. $\frac{2}{3}$
D. $-\frac{2}{3}$

Solution:
In the given equation $kx^2 + 6x + 4k = 0$
Sum of the roots = product of the roots (given)
$$-\frac{b}{a} = \frac{c}{a}$$
Here $a = k, b = 6$ and $c = 4k$
$$-\frac{6}{k} = \frac{4k}{k}$$
$k = -\frac{3}{2}$.

Q22. If $\sin\ a$ and $\cos\ a$ are the roots of the equation $ax^2 + bx + c = 0$, then $b^2 =$

A. $a^2 - 2ac$
B. $a^2 + 2ac$
C. $a^2 - ac$
D. $a^2 + ac$

Solution:
Equation $ax^2 + bx + c = 0$ has $\cos\ \alpha$ and $\cos\ \alpha$ as two roots
$$\sin\ a + \cos\ a = -\frac{b}{a}$$
$$\sin\ a \times \cos\ a = \frac{c}{a} \dots eq(1)$$
$$(\sin\ \alpha + \cos\ \alpha)^2 = \frac{b^2}{a^2}$$
$\sin^2\ \alpha + \cos^2\ \alpha + 2\sin\ \alpha . \cos\ \alpha = \frac{b^2}{a^2}$ .... eq (2)
But $\sin^2\ \alpha + \cos^2\ \alpha = 1$
$\therefore a^2(1 + 2\sin\ a \cdot \cos\ a) = b^2$

Putting $\sin\ a \times \cos\ a = \frac{c}{a}$, we get,

$\Rightarrow b^2 = a^2 + 2ac.$

Q23. If 2 is a root of the equation $x^2 + ax + 12 = 0$ and the quadratic equation $x^2 + ax + q = 0$ has equal roots, then q

A. 12

B. 8

C. 20

D. 16

Solution:

The given equation $x^2 + ax + 12 = 0$ has a root $= 2$

So it will satisfy the equation

$4 + 2a + 12 = 0$

$2a + 16 = 0$

$a = -8$

Putting value of a in second equation, it becomes

$x^2 + ax + q = 0$

$x^2 - 8x + q = 0$

Roots are equal so $d = 0$

$\Rightarrow b^2 - 4ac = 0$

$\Rightarrow 64 - 4q = 0$

$\Rightarrow q = \frac{64}{4}$

$\Rightarrow q = 16$

Q24. If the sum of the roots of the equation $x^2 - (k+6)x + 2(2k-1) = 0$ is equal to half of their product, then $k =$

A. 6

B. 7

C. 1

D. 5

Solution:

In the given equation $x^2 - (k+6)x + 2(2k-1) = 0$

$a = 1, b = -(k+6), c = 2(2k-1)$

Sum of the roots $= \frac{1}{2}$ (product of roots) (given)

$k + 6 = 2k - 1$

$k = 7.$

Q25. If a and b are roots of the equation $x^2 + ax + b = 0$, then $a + b =$

A. 1

B. 2

C. -2
D. -1
Solution:
Given a and b are roots of the equation $x^2+ax+b=0$
Here $a=1, b=a, c=b$
Sum of the roots $=a+b=-\frac{a}{1}$
$b=-2a$
Product of the roots
$ab=b$
$a=1$
$\therefore b=-2\times 1=-2$
Now $a+b=1+(-2)=-1$.

Q26. A quadratic equation whose one root is 2 and the sum of whose roots is zero, is
A. $x^2+4=0$
B. $x^2-4=0$
C. $4x^2-1=0$
D. $x^2-2=0$
Solution:
Let Root of an equation $=a=2$
Sum of the roots $=a+\beta=0$, where a and β are two roots of the equation
$\beta=-2$
$\alpha\beta=2x-2=-4$
the general equation is of the form
$x^2+(\text{ sum of the roots })x+\text{product of the roots}=0$
$x^2-4=0$ is the required equation.

Q27. If one root of the equation $ax^2+bx+c=0$ is three times the other, then $b^2:ac=$
A. 3: 1
B. 3: 16
C. 16: 3
D. 16: 1
Solution:
In the given equation $ax^2+bx+c=0$
Let a and β be the two roots
Given $a=3\beta$ ……..1
a $\beta=\frac{c}{a}$ (product of the roots)
$3\beta^2=\frac{c}{a}$cos. …. by using 1
$\beta^2=\frac{c}{3a}$…………… 2
$a+\beta=-\frac{b}{a}$

$$4\beta = -\frac{b}{a}$$

Squaring both the sides

$$16\beta^2 = \frac{b^2}{a^2}$$

By using 2

$$16 \times \frac{c}{3a} = \frac{b^2}{a^2}$$

$$\frac{16}{3} = \frac{b^2}{ac}.$$

Q28. If one root of the equation $2x^2 + kx + 4 = 0$ is 2, then the other root is

A. 6

B. -6

C. -1

D. 1

Solution:

In the given equation $2x^2 + kx + 4 = 0$

Let a and β be the two roots

$a = 2$ (given)

here $a = 2, b = k$ and $c = 4$

sum of the roots

$$\Rightarrow a + \beta = -\frac{b}{a}$$

$$\Rightarrow 2 + \beta = -\frac{k}{2}$$

$\beta = -k$

a $\beta = 2$

$\beta = \frac{2}{2} = 1$ (putting value of $a = 2$)

Q29. If one root of the equation $x^2 + ax + 3 = 0$ is 1, then its other root is

A. 3

B. -3

C. 2

D. -2

Solution:

Let the given equation has roots α and β

$a = 1$

Here $a = 1, b = a$ and $c = 3$

Sum of the roots

$$a + \beta = -\frac{b}{a} = -a$$

product of the roots

$$a\beta = \frac{c}{a} = 3$$
$$1\beta = 3$$
$$\beta = 3.$$

Q30. If one root of the equation $4x^2 - 2x + (\lambda - 4) = 0$ be the reciprocal of the other, then k

A. 8

B. -8

C. 4

D. -4

Solution:

Let a and β be the two roots of the given equation

$4x^2 - 2x + (\lambda - 4) = 0$

According to the given condition

$$a = \frac{1}{\beta}$$

Here $a = 4, b = -2$ and $c = (\lambda - 4)$

$$a + \beta = \frac{2}{4} = \frac{1}{2}$$
$$\frac{1}{\beta} + \beta = \frac{1}{2}$$
$$\text{a}\,\beta = \frac{k-4}{4}$$
$$\frac{1}{\beta}\beta = \frac{k-4}{4}$$
$$K = 8.$$

Q31. If $y = 1$ is a common root of the equations $ay^2 + ay + 3 = 0$ and $y^2 + y + b = 0$, then ab equals

A. 3

B. $-\frac{7}{2}$

C. 6

D. -3

Solution:

If $y = 1$ is root of both the equation it will staify both the equations

Putting $y = 1$ in first equation

$ay^2 + ay + 3 = 0$

$2a + 3 = 0$

$$a = -\frac{3}{2}$$

Putting value of y in second equation

$2 + b = 0$

$B = -2$

Now $ab = -\frac{3}{2}x - 2$

$ab = 3.$

Q32. The values of k for which the quadratic equation $16x^2 + 4kx + 9 = 0$ has real and equal roots.

A. $6, -\frac{1}{6}$

B. $36, -36$

C. $6, -6$

D. $\frac{3}{4}, -\frac{3}{4}$

Solution:

Given: $16x^2 + 4kx + 9 = 0$

To find:The values of k for which the quadratic equation $16x^2 + 4kx + 9 = 0$ has real and equal roots.

Solution:To have real and equal roots $d = 0$ Where $d = b^2 - 4ac$

$\Rightarrow b^2 - 4ac = 0$ Compare with the general equation of quadratic equation $ax^2 + bx + c = 0, a \neq 0$ here $a = 16, b = 4k$ and $c = 9$

$\Rightarrow b^2 - 4ac = (4k)^2 - 4 \times 16 \times 9 = 0$

$\Rightarrow 16k^2 - 576 = 0$

$\Rightarrow k^2 = \dfrac{576}{16}$

$\Rightarrow k = \dfrac{24}{4}$

$k = \pm 6$