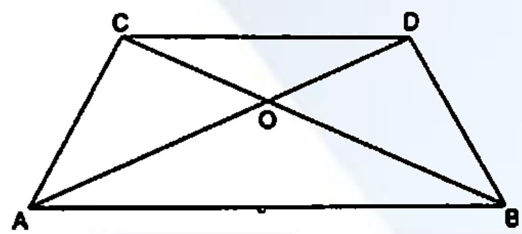
Solutions
Grade 08 Mathematics
Chapter 17: Understanding Shapes-III
(Special Types of Quadrilaterals)

Exercise 17.1

Q1. Given below is a parallelogram $ABCD$. Complete each statement along with the definition or property used.

(i) $AD =$
(ii) $\angle DCB =$
(iii) $OC =$
(iv) $\angle DAB + \angle CDA =$

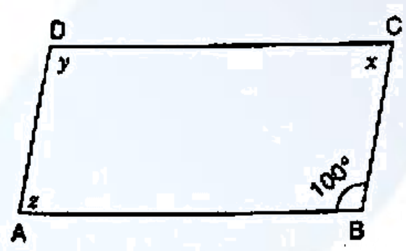


Solution:

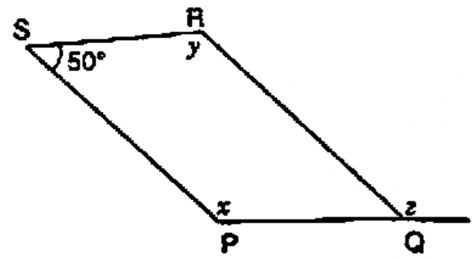
(i) $AD =$ BC [In a parallelogram diagonals bisect each other]
(ii) $\angle DCB = \angle BAD$ [alternate interior angles are equal]
(iii) $OC = OA$ [In a parallelogram diagonals bisect each other]
(iv) $\angle DAB + \angle CDA = 180°$ [Sum of adjacent angles in a parallelogram is 180°]

Q2. The following figures are parallelograms. Find the degree values of the unknowns x, y, z.

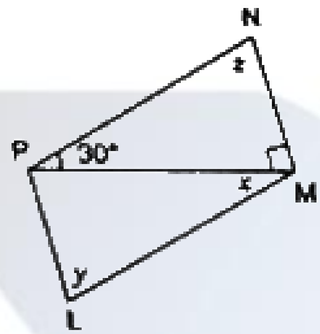
(i)



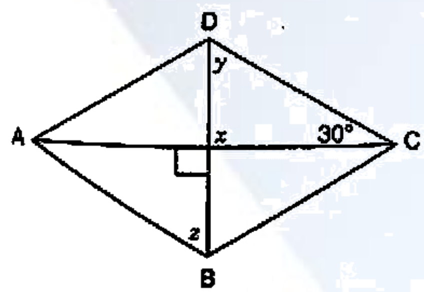
(ii)



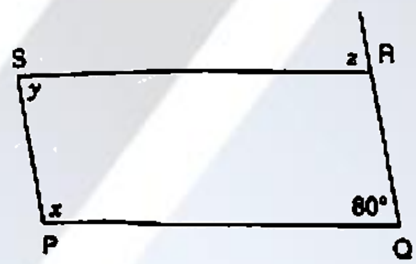
(iii)



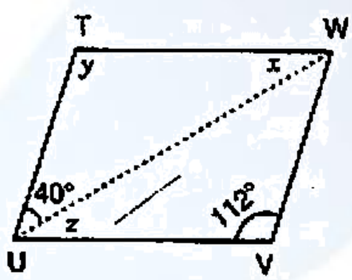
(iv)



(v)



(vi)



Solution:

(i) $\angle ABC = \angle y = 100^\circ$ [In a parallelogram opposite angles are equal]

$\angle x + \angle y = 180^\circ$ [In a parallelogram sum of the adjacent angles is equal to 180°]

$\angle x + 100^\circ = 180^\circ$

$\angle x = 180^\circ - 100^\circ$

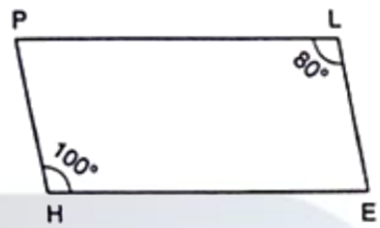
$\angle x = 80^\circ$

$\angle x = \angle z = 80^\circ$ [In a parallelogram opposite angles are equal]

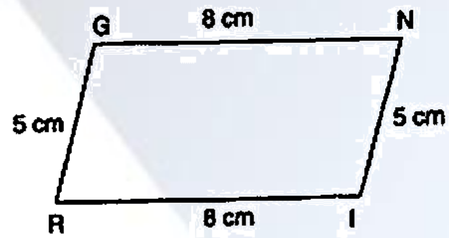
(ii) $\angle PSR + \angle y = 180^\circ$ [In a parallelogram sum of the adjacent angles is equal to 180°]

$\angle y + 50^\circ = 180^\circ$
$\angle y = 180^\circ - 50^\circ$
$\angle y = 130^\circ$
$\angle x = \angle y = 130^\circ$ [In a parallelogram opposite angles are equal]
$\angle PSR = \angle PQR = 50^\circ$ [In a parallelogram opposite angles are equal]
$\angle PQR + \angle z = 180^\circ$ [Linear pair]
$50^\circ + \angle z = 180^\circ$
$\angle z = 180^\circ - 50^\circ$
$\angle z = 130^\circ$
(iii) In ΔPMN
$\angle MPN + \angle PMN + \angle PNM = 180^\circ$ [Sum of all the angles of a triangle is 180°]
$30^\circ + 90^\circ + \angle z = 180^\circ$
$\angle z = 180^\circ - 120^\circ$
$\angle z = 60^\circ$
$\angle y = \angle z = 60^\circ$ [In a parallelogram opposite angles are equal]
$\angle z = 180^\circ - 120^\circ$ [In a parallelogram sum of the adjacent angles is equal to 180°]
$\angle z = 60^\circ$
$\angle z + \angle NML = 180^\circ$ [In a parallelogram sum of the adjacent angles is equal to 180°]
$60^\circ + 90^\circ + \angle x = 180^\circ$
$\angle x = 180^\circ - 150^\circ$
$\angle x = 30^\circ$
(iv) $\angle x = 90^\circ$ [vertically opposite angles are equal]
In $\triangle DOC$
$\angle x + \angle y + 30^\circ = 180^\circ$ [Sum of all the angles of a triangle is 180°]
$90^\circ + 30^\circ + \angle y = 180^\circ$
$\angle y = 180^\circ - 120^\circ$
$\angle y = 60^\circ$
$\angle y = \angle z = 60^\circ$ [alternate interior angles are equal]
(v) $\angle x + \angle POR = 180^\circ$ [In a parallelogram sum of the adjacent angles is equal to 180°]
$\angle x + 80^\circ = 180^\circ$
$\angle x = 180^\circ - 80^\circ$
$\angle x = 100^\circ$
$\angle y = 80^\circ$ [In a parallelogram opposite angles are equal]
$\angle QRS = \angle x = 100^\circ$
$\angle QRS + \angle z = 180^\circ$ [Linear pair]
$100^\circ + \angle z = 180^\circ$
$\angle z = 180^\circ - 100^\circ = 80^\circ$
(vi) $\angle y = 112^\circ$ [In a parallelogram opposite angles are equal]
$\angle y + \angle TUV = 180^\circ$ [In a parallelogram sum of the adjacent angles is equal to 180°]
$\angle z + 40^\circ + 112^\circ = 180^\circ$
$\angle z = 180^\circ - 152^\circ = 28^\circ$
$\angle z = \angle x = 28^\circ$ [alternate interior angles are equal]

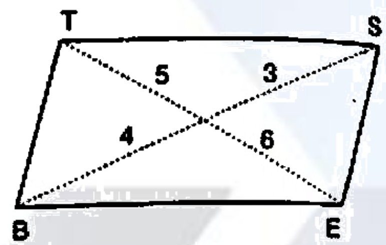
Q3. Can the following figures be parallelograms? Justify your Solution:
(i)



(ii)



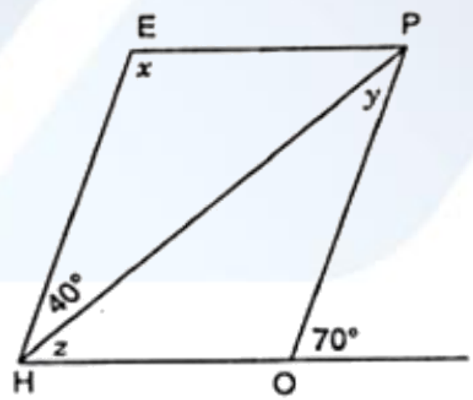
(iii)



Solution:

(i) No, In a parallelogram opposite angles are equal.
(ii) Yes, In a parallelogram opposite sides are equal and parallel.
(iii) No, In a parallelogram diagonals bisect each other.

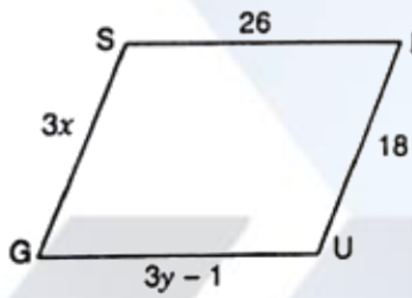
Q4. In the adjacent figure HOPE is a parallelogram. Find the angle measures x, y and z. State the geometrical truths you use to find them.



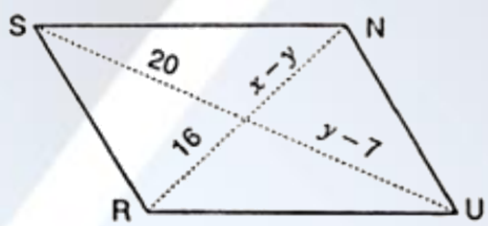
Solution:

$\angle HOP + 70^\circ = 180^\circ$ [Linear pair]
$\angle HOP = 180^\circ - 70^\circ$
$\angle HOP = 110^\circ$
$\angle HOP = \angle x = 110^\circ$ [In a parallelogram opposite angles are equal]
$\angle x + \angle z + 40^\circ = 180^\circ$ [In a parallelogram sum of the adjacent angles is equal to 180°]
$110^\circ + \angle z + 40^\circ = 180^\circ$
$\angle z = 180^\circ - 150^\circ$
$\angle z = 30^\circ$
$\angle z + \angle y = 70^\circ$
$\angle y + 30^\circ = 70^\circ$
$\angle y = 70^\circ - 30^\circ$
$\angle y = 40^\circ$

Q5. In the following figures GUNS and RUNS are parallelograms. Find x and y.
(i)



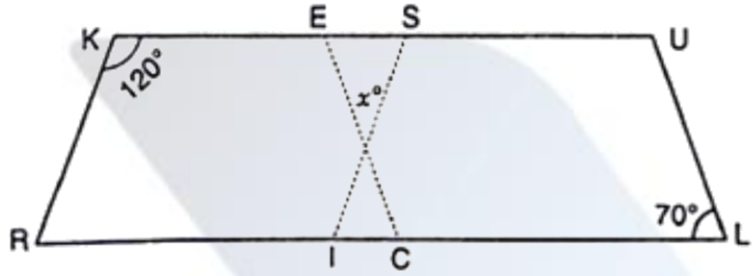
(ii)



Solution:
(i) $3y - 1 = 26$ [In a parallelogram opposite sides are of equal length]
$3y = 26 + 1$
$y = \frac{27}{3} = 9$
$y = 9$ units
$3x = 18$ [In a parallelogram opposite sides are of equal length]
$x = \frac{18}{3} = 6$
$x = 6$ units
(ii) $y - 7 = 20$ [In a parallelogram diagonals bisect each other]
$y = 20 + 7$
$y = 27$ units
$x - y = 16$ [In a parallelogram diagonals bisect each other]

$x - 27 = 16$
$x = 16 + 27 = 43$ units

Q6. In the following figure RISK and CLUE are parallelograms. Find the measure of x.



Solution:

In parallelogram RISK

$\angle SKR + \angle ISK = 180^{\circ}$ [In a parallelogram sum of the adjacent angles is equal to 180°]

$120^{\circ} + \angle ISK = 180^{\circ}$

$\angle ISK = 180^{\circ} - 120^{\circ}$

$\angle z = 60^{\circ}$

In parallelogram CLUE

$\angle UEC = \angle z = 70^{\circ}$ [In a parallelogram opposite angles are equal]

In $\triangle$ EOS

$70^{\circ} + \angle x + 60^{\circ} = 180^{\circ}$ [Sum of angles of a triangles is 180°]

$\angle x = 180^{\circ} - 130^{\circ}$

$\angle x = 50^{\circ}$

Q7. Two opposite angles of a parallelogram are $(3x - 2)^{\circ}$ and $(50 - x)^{\circ}$. Find the measure of each angle of the parallelogram.

Solution:

We know that opposite angles of a parallelogram are equal.

Therefore $(3x - 2)^{\circ} = (50 - x)^{\circ}$

$3x - 2^{\circ} = 50^{\circ} - x$

$3x^{\circ} + x = 50^{\circ} + 2^{\circ}$

$4x = 52^{\circ}$

$x = \frac{52^{\circ}}{4} = 13^{\circ}$

Measure of opposite angles are: $3x - 2 = 3 \times 13^{\circ} - 2 = 37^{\circ}$

$(50 - x)^{\circ} = 50 - 13 = 37^{\circ}$

Sum of adjacent angles $= 180^{\circ}$

Other two angles are $180^{\circ} - 37^{\circ} = 143^{\circ}$ each.

Q8. If an angle of a parallelogram is two-third of its adjacent angle, find the angles of the parallelogram.

Solution:

Let one of the adjacent angle is x°
Therfore other adjacent angle $= \frac{2x^\circ}{3}$
Sum of adjacent angles $= 180^\circ$
$$x^\circ + \frac{2x^\circ}{3} = 180^\circ$$
$$\frac{3x^\circ + 2x^\circ}{3} = 180^\circ$$
$$\frac{5x^\circ}{3} = 180^\circ$$
$$x^\circ = \frac{180^\circ \times 3}{5}$$
$$x^\circ = 108^\circ$$
Other angle $= 180^\circ - 108^\circ = 72^\circ$
Therefore angles of the parallelograms are $72^\circ, 72^\circ, 108^\circ$ and 108°

Q9. The measure of one angle of a parallelogram is 70°. What are the measures of the remaining angles?

Solution:

Let one of the adjacent angle is x°
Therfore other adjacent angle $= 70^\circ$
Sum of adjacent angles $= 180^\circ$
$$x^\circ + 70^\circ = 180^\circ$$
$$x^\circ = 180^\circ - 70^\circ$$
$$x^\circ = 110^\circ$$
Therefore angles of the parallelograms are $70^\circ, 70^\circ, 110^\circ$ and 110°

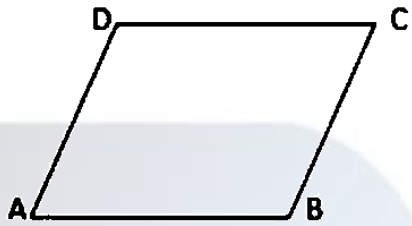
Q10. Two adjacent angles of a parallelogram are as $1:2$. Find the measures of all the angles of the parallelogram.

Solution:

Let one of the adjacent angles are x°
Therfore adjacent angles are $= x^\circ$ and $2x^\circ$
Sum of adjacent angles $= 180^\circ$
$$x^\circ + 2x^\circ = 180^\circ$$
$$3x^\circ = 180^\circ$$
$$x^\circ = \frac{180^\circ}{3}$$
$$x^\circ = 60^\circ$$
Other angle $= 180^\circ - 60^\circ = 120^\circ$
Therefore angles of the parallelograms are $60^\circ, 60^\circ, 120^\circ$ and 120°

Q11. In a parallelogram $ABCD, \angle D = 135^\circ$, determine the measure of $\angle A$ and $\angle B$.

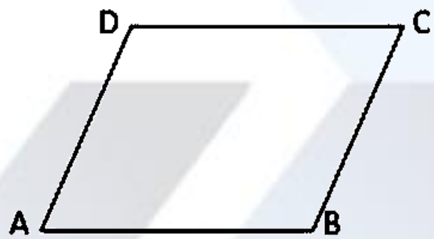
Solution:



Let one of the adjacent angle $\angle D = 135^\circ$
Therfore other adjacent angle $\angle A = x^\circ$
Sum of adjacent angles $= 180^\circ$
$x^\circ + 135^\circ = 180^\circ$
$x^\circ = 180^\circ - 135^\circ$
$\angle A = x^\circ = 45^\circ$
$\angle A = \angle C = 45^\circ$ and $\angle D = \angle B = 135^\circ$ [Measure of opposite angles of a parallelogram are equal]

Q12. $ABCD$ is a parallelogram in which $\angle A = 70^\circ$. Compute $\angle B, \angle C$ and $\angle D$.

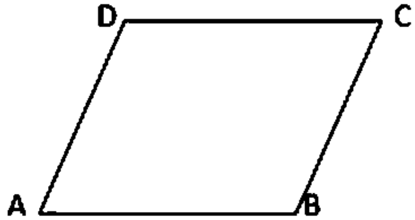
Solution:



Let one of the adjacent angle $\angle A = 70^\circ$
Therfore other adjacent angle $\angle B = \mathrm{x}^\circ$
Sum of adjacent angles $= 180^\circ$
$x^\circ + 70^\circ = 180^\circ$
$x^\circ = 180^\circ - 70^\circ$
$\angle B = x^\circ = 110^\circ$
$\angle A = \angle C = 70^\circ$ and $\angle D = \angle B = 110^\circ$ [Measure of opposite angles of a parallelogram are equal]

Q13. The sum of two opposite angles of a parallelogram is 130°. Find all the angles of the parallelogram.

Solution:



Let one of the adjacent angle $\angle A = 130^\circ$
Therfore other adjacent angle $\angle B = \text{x}^\circ$
Sum of adjacent angles $= 180^\circ$
$x^\circ + 130^\circ = 180^\circ$
$x^\circ = 180^\circ - 130^\circ$
$\angle B = x^\circ = 50^\circ$
$\angle A = \angle C = 130^\circ$ and $\angle D = \angle B = 70^\circ$ [Measure of opposite angles of a parallelogram are equal]

Q14. All the angles of a quadrilateral are equal to each other. Find the measure of each. Is the quadrilateral a parallelogram? What special type of parallelogram is it?

Solution:

Let each angle of parallelogram ABCD = x°
Sum of all the angles $= 360^\circ$
$x^\circ + x^\circ + x^\circ + x^\circ = 360^\circ$
$4x^\circ = 360^\circ$
$x^\circ = \frac{360^\circ}{4} = 90^\circ$
Therefore, each angle of the parallelogram is equal to 90°
Yes, this quadrilateral is a parallelogram. Aparallelogram with each angle equal to 90° is a rectangle.

Q15. Two adjacent sides of a parallelogram are 4 cm and 3 cm respectively. Find its perimeter.

Solution:

We know that opposite sides of a parallelogram are equal and parallel.
Perimeter = Sum of all sides
Perimeter $= 4 + 3 + 4 + 3 = 14$ cm

Q16. The perimeter of a parallelogram is 150 cm. One of its sides is greater than the other by 25 cm. Find the length of the sides of the parallelogram.

Solution:

Perimeter of the parallelogram = 150 cm
Let one of the sides $= x$ cm
Other side $= (x + 25)$cm
We know that opposite sides of a parallelogram are equal and parallel.
Perimeter = Sum of all sides
$x + x + 25 + x + x + 25 = 150$
$4x + 50 = 150$

$4x = 150 - 50$
$x = \frac{100}{4} = 25$
Therefore sides of the parallelogram are 25 cm and 50 cm .

Q17. The shorter side of a parallelogram is 4.8 cm and the longer side is half as much again as the shorter side. Find the perimeter of the parallelogram.

Solution:

Shorter side of the parallelogram $= 4.8$ cm
Longer side of the parallelogram $= 4.8 + \frac{4.9}{2}$
$= 4.8 + 2.4 = 7.2$ cm
We know that opposite sides of a parallelogram are equal and parallel.
Perimeter $=$ Sum of all sides
Perimeter of the parallelogram $= 4.8 + 7.2 + 4.8 + 7.2 = 24$ cm
Therefore perimeter of the parallelogram 24 cm .

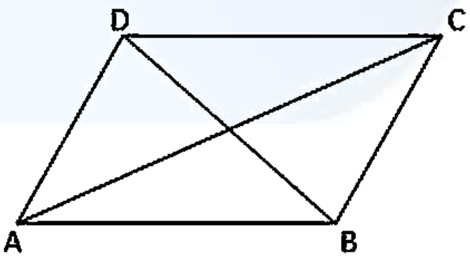
Q18. Two adjacent angles of a parallelogram are $(3x - 4)^\circ$ and $(3x + 10)^\circ$. Find the angles of the parallelogram.

Solution:

We know that sum of the adjacent angles of a parallelogram $= 180^\circ$
$(3x - 4)^\circ + (3x + 10)^\circ = 180^\circ$
$3x^\circ - 4^\circ + 3x^\circ + 10^\circ = 180^\circ$
$6x^\circ = 180^\circ - 6^\circ$
$x = \frac{174^\circ}{6} = 29^\circ$
Measure of one angle: $3x - 4 = 3 \times 29^\circ - 4 = 83^\circ$
Measure of other angle $= (3x + 10)^\circ = 3 \times 29 + 10 = 97^\circ$
Therefore angles of the parallelogram are $83^\circ 83^\circ, 97^\circ$ and 97°

Q19. In a parallelogram $ABCD$, the diagonals bisect each other at O. If $\angle ABC = 30^\circ, \angle BDC = 10^\circ$ and $\angle CAB = 70^\circ$. Find: $\angle DAB, \angle ADC, \angle BCD, \angle AOD, \angle DOC, \angle BOC, \angle AOB, \angle ACD, \angle CAB, \angle ADB, \angle ACB, \angle DBC$, and $\angle DBA$.

Solution:



$\angle ABC = \angle ADC = 30^\circ$ [Measure of opposite angles is equal in a parallelogram]
$\angle BDC = 10^\circ$ given
$\angle BDA = 30^\circ - 10^\circ = 20^\circ$
$\angle DAB = 180^\circ - 30^\circ = 150^\circ$

$\angle BCD = \angle DAB = 150°$ [Measure of opposite angles is equal in a parallelogram]
$\angle DBA = \angle BDC = 10°$ [Alternate interior angles are equal]
In $\triangle DOC$
$\angle BDC + \angle ACD + \angle DOC = 180°$ [Sum of all angles og a triangle is 180°]
$10° + 70° + \angle DOC = 180°$
$\angle DOC = 180° - 80°$
$\angle DOC = 100°$
$\angle DOC = \angle AOB = 100°$ [Vertically opposite angles are equal]
$\angle DOC + \angle AOD = 180°$[Linear pair]
$100° + \angle AOD = 180°$
$\angle AOD = 180° - 100°$
$\angle AOD = 80°$
$\angle AOD = \angle BOC = 80°$ [Vertically opposite angles are equal]
$\angle ABC + \angle BCD = 180°$ [In a parallelogram sum of adjacent angles is 180°]
$30° + \angle ACB + \angle ACD = 180°$
$30° + \angle ACB + 70° = 180°$
$\angle ACB = 180° - 100°$
$\angle ACB = 80°$
$\angle ACB = \angle ACB = 80°$ [Alternate interior angles are equal]

Q20. Find the angles marked with a question mark shown in Fig. 17.27

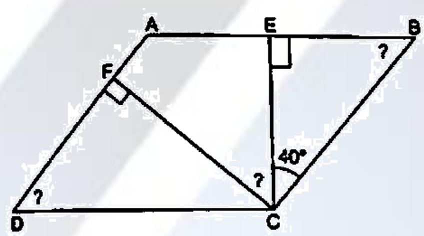


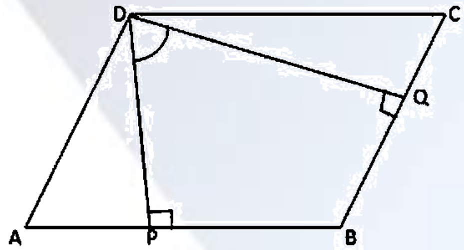
Fig. 17.27

Solution:
In $\triangle BEC$
$\angle BEC + \angle ECB + \angle CBC = 180°$ [Sum of angles of a triangle is 180°]
$90° + 40° + \angle CBC = 180°$
$\angle CBC = 180° - 130°$
$\angle CBC = 50°$
$\angle B = \angle D = 50°$ [Opposite angles of a parallelogram are equal]
$\angle A + \angle B = 180°$ [Sum of adjacent angles of a triangle is 180°]
$\angle A + 50° = 180°$
$\angle A = 180° - 50°$
$\angle A = 130°$
In $\triangle$ DFC
$\angle DFC + \angle FCD + \angle CDF = 180°$ [Sum of angles of a triangle is 180°]
$90° + \angle FCD + 50° = 180°$
$\angle FCD = 180° - 140°$
$\angle FCD = 40°$

$\angle A = \angle C = 130^\circ$ [Opposite angles of a parallelogram are equal]
$\angle C = \angle FCE + \angle BCE + \angle FCD$
$\angle DCF + 40^\circ + 40^\circ = 130^\circ$
$\angle DCF = 130^\circ - 80^\circ$
$\angle DCF = 50^\circ$

Q21. The angle between the altitudes of a parallelogram, through the same vertex of an obtuse angle of the parallelogram is 60°. Find the angles of the parallelogram.

Solution:



Given $ABCD$ is a parallelogram in which $DP \perp AB$ and $AQ \perp BC$.
Given $\angle PDQ = 60^\circ$
In quad. $DPBQ$
$\angle PDQ + \angle DPB + \angle B + \angle BQD = 360^\circ$ [Sum of all the angles of a Quad is 360°]
$60^\circ + 90^\circ + \angle B + 90^\circ = 360^\circ$
$\angle B = 360^\circ - 240^\circ$
Therefore, $\angle B = 120^\circ$
But $\angle B = \angle D = 120^\circ$ [Opposite angles of parallelogram are equal]
$\angle B + \angle C = 180^\circ$ [Sum of adjacent interior angles in a parallelogram is 180°]
$120^\circ + \angle C = 180^\circ$
$\angle C = 180^\circ - 120^\circ = 60^\circ$
Therefore, $\angle A = \angle C = 70^\circ$ (Opposite angles of parallelogram are equal)

Q22. In Fig. 17.28, $ABCD$ and $AEFG$ are parallelograms. If $\angle C = 55^\circ$, what is the measure of $\angle F$? Figure

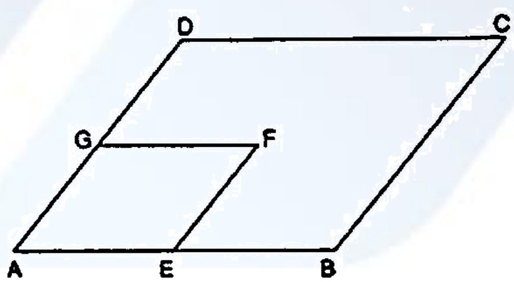


Fig. 17.28

Solution:

In parallelogram ABCD
$\angle C = \angle A = 55^\circ$ [In a parallelogram opposite angles are equal]
In parallelogram AEFG
$\angle A = \angle F = 55^\circ$ [In a parallelogram opposite angles are equal]

Q23. In Fig. 17.29, $BDEF$ and $DCEF$ are each a parallelogram. Is it true that $BD = DC$? Why or why not?

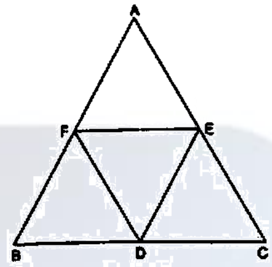


Fig. 17.29

Solution:

In parallelogram BDEF

$BD = EF$ (i) [In a parallelogram opposite sides are equal]

In parallelogram DCEF

$DC = EF$ (ii) [In a parallelogram opposite sides are equal]

From equations (i) and (ii), we get

$BD = EF = DC$

Hence, BD = DC Proved

Q24. In Fig. 17.29, suppose it is known that $DE = DF$. Then, is $\triangle ABC$ isosceles? Why or why not?

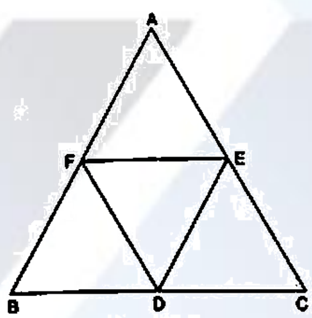


Fig. 17.29

Solution:

In parallelogram BDEF

$BD = EF$ and $BF = DE$ … ......(i) [In a parallelogram opposite sides are equal]

In parallelogram DCEF

$DC = EF$ and $DF = CE$ … .... (ii) [In a parallelogram opposite sides are equal]

In parallelogram AFDE

$AF = DE$ and $DF = AE$ (ii) [In a parallelogram opposite sides are equal]

Therefore $DE = AF = BF$

Similarly: $DF = CE = AE$

But, DE = DF given

From equations (iv) and (v), we get

$AF + BF = AE + EC$

$AB = AC$

Therefore $\triangle$ ABC is an isosceles triangle.

Q25. Diagonals of parallelogram $ABCD$ intersect at O as shown in Fig. 17.30. XY contains O, and X, Y are points on opposite sides of the parallelogram. Give reasons for each of the following:

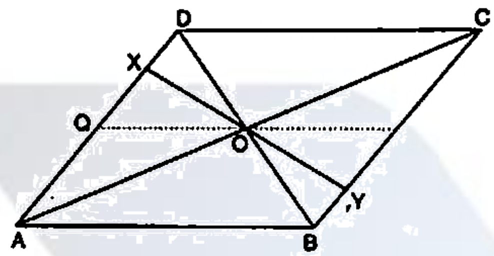


Fig. 17.30

Solution:

(i) $OB = OD$

$OB = OD$ [In a parallelogram diagonals bisect each other]

(ii) $\angle OBY = \angle ODX$ [Alternate interior angles are equal]

(iii) $\angle BOY = \angle DOX$ [Vertically opposite angles are equal]

(iv) $\triangle BOY \cong \triangle DOX$

In $\triangle BOY$ and $\triangle DOX$

$OB = OD$ [In a parallelogram diagonals bisect each other]

$\angle OBY = \angle ODX$ [Alternate interior angles are equal]

$\angle BOY = \angle DOX$ [Vertically opposite angles are equal]

$\triangle BOY \cong \triangle DOX$ [ASA rule]

Now, state if XY is bisected at O.

Hence OX = OY [Corresponding parts of congruent triangles]

Q26. In Fig. 17.31, $ABCD$ is a parallelogram, CE bisects $\angle C$ and AF bisects $\angle A$. In each of the following, if the statement is true, give a reason for the same.

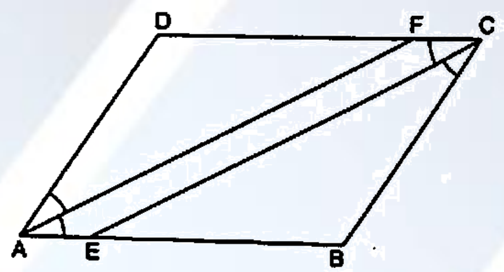


Fig. 17.31

Solution:

(i) $\angle A = \angle C$

True,

$\angle C = \angle A = 55°$ [In a parallelogram opposite angles are equal]

(ii) $\angle FAB = \frac{1}{2}\angle A$

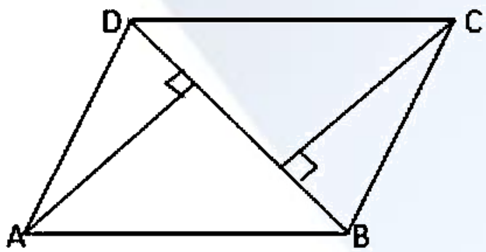
True,

AF is the angle bisector of angle A

(iii) $\angle DCE = \frac{1}{2}\angle C$

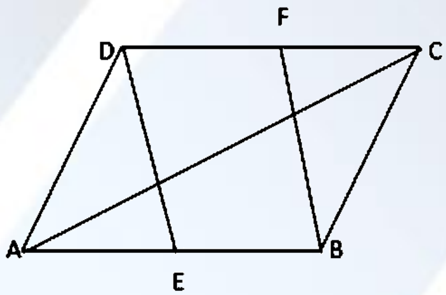
True,
CE is the angle bisectoe of angle A
(iv) $\angle CEB = \angle FAB$
True,
$\angle C = \angle A$ [In a parallelogram opposite angles are equal]
$\frac{1}{2}\angle C = \frac{1}{2}\angle A$ [AF and CE are angle bisectors]
(v) $CE \| AF$
True, Since one pair of opposite angles are equal, therefore Quad AEFC is a parallelogram.

Q27. Diagonals of a parallelogram $ABCD$ intersect at O. AL and CM are drawn perpendiculars to BD such that L and M lie on BD. Is $AL = CM$? Why or why not?
Solution:



AL and CM are perpendiculars on diagonal BD.
$AL = CM$ [In a parallelogram length of perpendiculars drawn on diagonal from opposite vertices are equal]

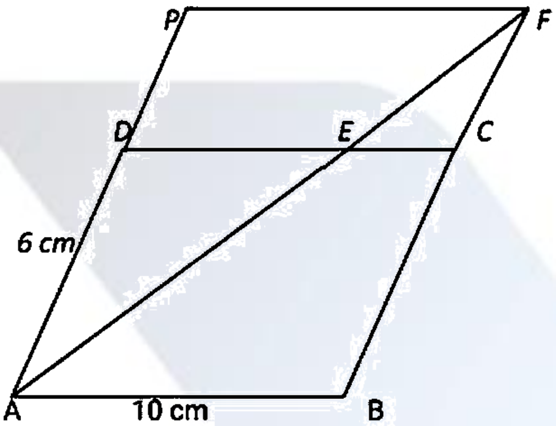
Q28. Points E and F lie on diagonals AC of a parallelogram $ABCD$ such that $AE = CF$. What type of quadrilateral is $BFDE$?
Solution:



In parallelogram $ABCD$:
$AB = CD$ (i) [In aparallelogram opposite sides are equal and parallel]
$AE = CF$ (ii) given
On subtracting (ii) from (i)
$AB - AE = CD - CF$
$BE = DF$
BE parallel to DF
Therefore quad BFDE is a parallelogram, since one pair of opposite sides are equal and parallel.

Q29. In a parallelogram $ABCD, AB = 10$ cm, $AD = 6$ cm. The bisector of $\angle A$ meets DC in E, AE and BC produced meet at F. Find the length CF.

Solution:



In a parallelogram $ABCD$

$AB = 10$ cm, $AD = 6$ cm

$\Rightarrow DC = AB = 10$ cm and $AD = BC = 6$ cm [In a parallelogram opposite sides are equal]

Given that bisector of $\angle A$ intersects DE at E and BC produced at F.

Draw PF || CD

From the figure, CD || FP and CF || DP

Hence PDCF is a parallelogram. [Since one pair of opposite sides are equal and parallel]

AB || FP and $AP|| BF$

$\Rightarrow ABFP$ is also a parallelogram

Consider $\triangle APF$ and $\triangle ABF$

$\angle APF = \angle ABF$ [Since opposite angles of a parallelogram are equal]

$AF = AF$ (Common side)

$\angle PAF = \angle AFB$ (Alternate angles)

$\triangle APF \cong \triangle ABF$ (By ASA congruence criterion)

$\Rightarrow AB = AP(CPCT)$

$\Rightarrow AB = AD + DP = AD + CF$ [Since DCFP is a parallelogram]

$\therefore CF = AB -$ADCF $= (10 - 6)$cm $= 4$ cm

Exercise 17.2

Q1. Which of the following statements are true for a rhombus?

(i) It has two pairs of parallel sides.
(ii) It has two pairs of equal sides.
(iii) It has only two pairs of equal sides.
(iv) Two of its angles are at right angles.
(v) Its diagonals bisect each other at right angles.
(vi) Its diagonals are equal and perpendicular.
(vii) It has all its sides of equal lengths.
(viii) It is a parallelogram.
(ix) It is a quadrilateral.
(x) It can be a square.
(xi) It is a square.

Solution:

(i) True, Rhombus is a parallelogram.
(ii) True, Rhombus has all four sides equal.
(iii) False, Rhombus has all four sides equal.
(iv) False, In rhombus no angle is right angle.
(v) True, in rhombus diagonals bisect each other at right angles.
(vi) False, in rhombus diagonals are of unequal length.
(vii) True, Rhombus has all four sides equal.
(viii) True, Rhombus is a parallelogram since opposite sides equal and parallel.
(ix) True, Rhombus is a quadrilateral since it has four sides.
(x) True, Rhombus becomes square when any one angle is 90°.
(xi) False, Rhombus is never a square. Since in a square each angle is 90°.

Q2. Fill in the blanks, in each of the following, so as to make the statement true:

(i) A rhombus is a parallelogram in which……………………..
(ii) A square is a rhombus in which…………………………..
(iii) A rhombus has all its sides of length……………………..
(iv) The diagonals of a rhombus   each other at angles……………………..
(v) If the diagonals of a parallelogram bisect each other at right angles, then it is a ………………………………..

Solution:

(i) A rhombus is a parallelogram in which opposite sides are equal and parallel.
(ii) A square is a rhombus in which all four sides are equal.
(iii) A rhombus has all its sides of equal length.
(iv) The diagonals of a rhombus bisect each other at right angles.
(v) If the diagonals of a parallelogram bisect each other at right angles, then it is a rhombus.

Q3. The diagonals of a parallelogram are not perpendicular. Is it a rhombus? Why or why not?

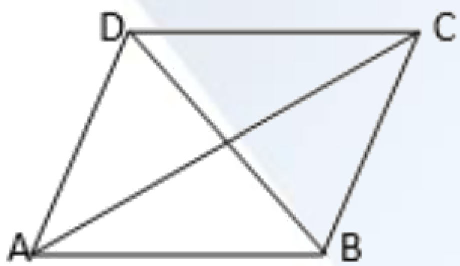
Solution:

No, Diagonals of a rhombus bisect each other at 90°.
A parallelogram is rhombus only when its diagonals bisect each other at right angles.

Q4. The diagonals of a quadrilateral are perpendicular to each other. Is such a quadrilateral always a rhombus? If your Solution is ' No ', draw a figure to justify your solution.

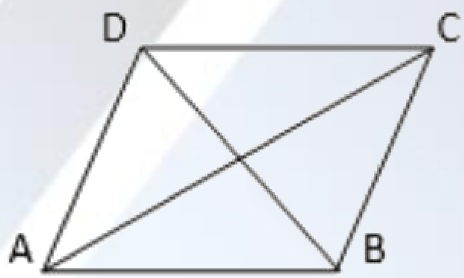
Solution:

No, it is not always a rhombus.



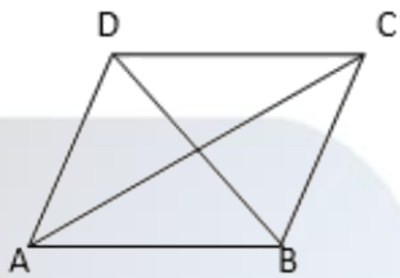
Q5. $ABCD$ is a rhombus. If $\angle ACB = 40^\circ$, find $\angle ADB$.

Solution:



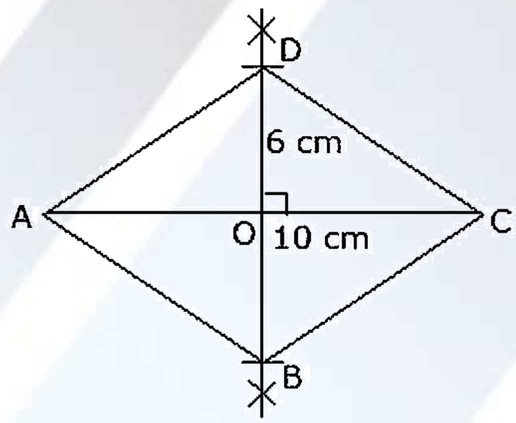
In rhombus ABCD
$\angle ACB = 40^\circ$ given
$\angle ACB = \angle CAD = 40^\circ$ [Altermate interior angles are equal]
In $\triangle AOD$
$\angle AOD = 90^\circ$ [In rhombus diagonals bisect eact other at right angles]
$\angle AOD + \angle CAD + \angle ADB = 180^\circ$ [Angle sum property of a triangle]
$90^\circ + 40^\circ + \angle ADB = 180^\circ$
$\angle ADB = 180^\circ - 130^\circ$
$\angle ADB = 50^\circ$

Q6. If the diagonals of a rhombus are 12 cm and 16 cm , find the length of each side.
Solution:



We know in rhombus diagonals bisect each other at right angle.
In △ AOB
$AO = \frac{12}{2} = 6 \text{ cm}, \text{BO} = \frac{16}{2} = 8 \text{ cm}$
Using Pythagoras theorem in $\triangle AOB$
$AB^2 = AO^2 + BO^2$
$AB^2 = 6^2 + 8^2$
$AB^2 = 36 + 64$
$AB^2 = 100$
$AB = \sqrt{100} = 10 \text{ cm}$
Therefore each side of a rhombus is 10 cm.

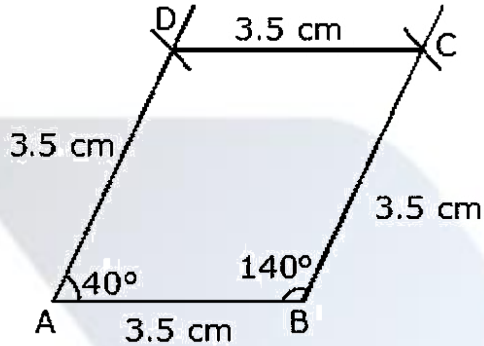
Q7. Construct a rhombus whose diagonals are of length 10 cm and 6 cm.
Solution:



Steps of Construction:
(i) Draw diagonal AC of length 10 cm .
(ii) Draw perpendicular bisector of AC at point O .
(iii) From point ' O ' out two arcs of length 3 cm to get points B and D .
(iv) Join AD and DC to get rhombus $ABCD$.

Q8. Draw a rhombus, having each side of length 3.5 cm and one of the angles as 40°.

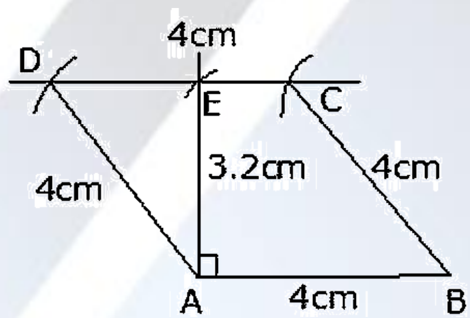
Solution:



Steps of construction:
(i) Draw a line segment AB of length 3.5 cm
(ii) From point A and B draw angles of 40 and 140 respectively.
(iii) From points A and B draw two arcs of length 3.5 cm each is get points D and C.
(iv) Join ABCD to get rhombus ABCD.

Q9. One side of a rhombus is of length 4 cm and the length of an altitude is 3.2 cm . Draw the rhombus.

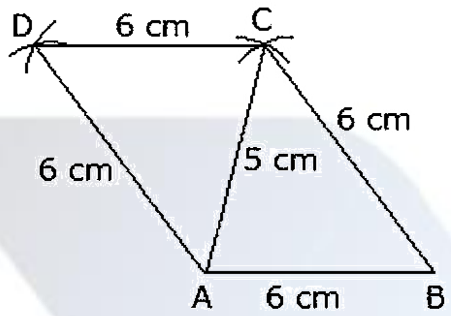
Solution:



Steps of construction:
(i) Draw a line segment of 4 cm
(ii) From point A draw a perpendicular from point A and cut a length of 3.2 cm to get point E .
(iii) From point E and a line parallel to AB .
(iv) From points A and B cut two arcs of length 4 cm on the drawn parallel line to get points D and C.
(v) Join AD and BC to get rhombus $ABCD$.

Q10. Draw a rhombus $ABCD$, if $AB = 6$ cm and $AC = 5$ cm.

Solution:



Steps of construction:

(i) Draw a line segment AB of length 6 cm.

(ii) From point ' A ' draw an arc of length 5 cm and from point B draw an arc of length 6 cm . Such that both the arcs intersect at ' C '.

(iii) Join AC and BC.

(iv) From point A draw an arc of length 6 cm and from point C draw an arc of 6 cm , so that both the arcs intersect at point D.

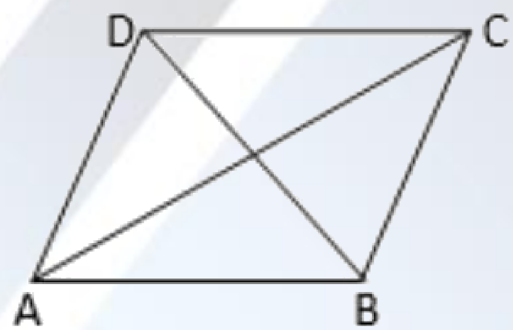
(v) Joint AD and DC to get rhombus ABCD.

Q11. $ABCD$ is a rhombus and its diagonals intersect at O.

(i) Is $\triangle BOC \cong \triangle DOC$? State the condruence condition used?

(ii) Also state, if $\angle BCO = \angle DCO$.

Solution:



(i) In $\triangle BOC$ and $\triangle DOC$

BO = DO [In a rhombus diagonals bisect each other]

CO = CO Common

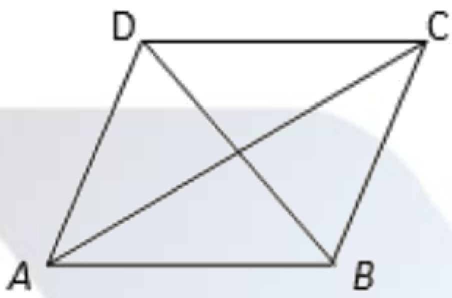
$BC = CD$[All sides of a rhombus are equal]

$\triangle BOC \cong \triangle DOC$ [SSS Congurency]

(ii) $\angle BCO = \angle DCO$ from above [corresponding parts of congruent triangles]

Q12. Show that each diagonal of a rhombus bisects the angle through which it passes.

Solution:



In $\triangle BOC$ and $\triangle DOC$

BO $=$ DO [In a rhombus diagonals bisect each other]

CO $=$ CO Common

$BC = CD$[All sides of a rhombus are equal]

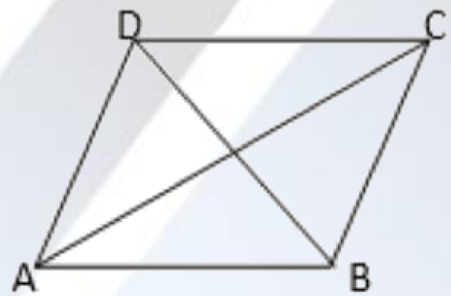
$\triangle BOC \cong \triangle DOC$ [SSS Congurency]

$\angle BCO = \angle DCO$ from above [corresponding parts of congruent triangles]

Hence, each diagonal of a rhombus bisect the angle through which it passes.

Q13. $ABCD$ is a rhombus whose diagonals interesct at O. If $AB = 10$ cm, diagonal $BD = 16$ cm, find the length of diagonal AC.

Solution:



We know in rhombus diagonals bisect each other at right angle.

In $\triangle AOB$

$BO = \frac{BD}{2} = \frac{16}{2} = 8$ cm

Using Pythagoras theorem in $\triangle AOB$

$AB^2 = AO^2 + BO^2$

$10^2 = AO^2 + 8^2$

$100 - 64 = AO^2$

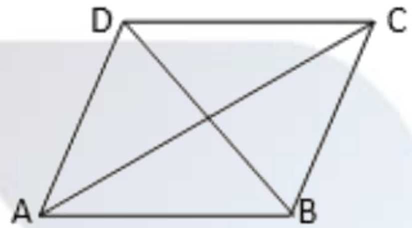
$AO^2 = 36$

$AO = \sqrt{36} = 6$ cm

Therefore length of diagonal AC of rhombus ABCD is $6 \times 2 = 12$ cm.

Q14. The diagonal of a quadrilateral are of lengths 6 cm and 8 cm . If the diagonals bisect each other at right angles, what is the length of each side of the quadrilateral?

Solution:



We know in rhombus diagonals bisect each other at right angle.

In $\triangle AOB$

$\mathrm{BO} = \frac{BD}{2} = \frac{6}{2} = 3 \text{ cm}$

$AO = \frac{AC}{2} = \frac{8}{2} = 4 \text{ cm}$

Using Pythagoras theorem in $\triangle AOB$

$AB^2 = AO^2 + BO^2$

$AB^2 = 4^2 + 3^2$

$AB^2 = 16 + 9$

$AB^2 = 25$

$AB = \sqrt{25} = 6 \text{ cm}$

Therefore length of each side of a rhombus $ABCD$ is 5 cm .

Exercise 17.3

Q1. Which of the following statements are true for a rectangle?

(i) It has two pairs of equal sides.
(ii) It has all its sides of equal length.
(iii) Its diagonals are equal.
(iv) Its diagonals bisect each other.
(v) Its diagonals are perpendicular.
(vi) Its diagonals are perpendicular and bisect each other.
(vii) Its diagonals are equal and bisect each other.
(viii) Its diagonals are equal and perpendicular, and bisect each other.
(ix) All rectangles are squares.
(x) All rhombuses are parallelograms.
(xi) All squares are rhombuses and also rectangles.
(xii) All squares are not parallelograms.

Solution:

(i) True, In a rectangle two pairs of sides are equal.
(ii) False, In a rectangle two pairs of sides are equal.
(iii) True, In a rectangle diagonals are of equal length.
(iv) True, In a rectangle diagonals bisect each other.
(v) False, Diagonals of a rectangle need not be perpendicular.
(vi) False, Diagonals of a rectangle need not be perpendicular. Diagonals only

bisect each other.
(vii) True, Diagonals are of equal length and bisect each other.
(viii) False, Diagonals are of equal length and bisect each other. Diagonals of a rectangle need not be perpendicular
(ix) False, In a square all sides are of equal length.
(x) True, All rhombuses are parallelograms, since opposite sides are equal and parallel.
(xi) True, All squares are rhombuses, since all sides are equal in a square and rhombus. All squares are rectangles, since opposite sides are equal and parallel.
(xii) False, All squares are parallelograms, since opposite sides are parallel and equal.

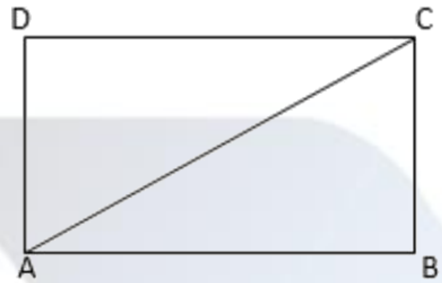
Q2. Which of the following statements are true for a square?
(i) It is a rectangle.
(ii) It has all its sides of equal length.
(iii) Its diagonals bisect each other at right angle.
(v) Its diagonals are equal to its sides.
Solution:
(i) True, square is a rectangle, since opposite sides are equal and parallel and each angle is right angle.
(ii) True, In a square all sides are of equal length.
(iii) True, in a square diagonals bisect each other at right angle.
(v) False, in a square diagonals are of equal length. Length of diagonals is not equal to the length of sides

Q3. Fill in the blanks in each of the following, so as to make the statement true :
(i) A rectangle is a parallelogram in which .
(ii) A square is a rhombus in which .
(iii) A square is a rectangle in which .
Solution:
(i) A rectangle is a parallelogram in which opposite sides are parallel and equal.
(ii) A square is a rhombus in which all the sides are of equal length.
(iii) A square is a rectangle in which opposite sides are equal and parallel and each angle is a right angle.

Q4. A window frame has one diagonal longer then the other. Is the window frame a rectangle? Why or why not?
Solution:
No, diagonals of a rectangle are of equal length equal.

Q5. In a rectangle $ABCD$, prove that $\triangle ACB \cong \triangle CAD$.

Solution:



In $\triangle ACB$ and $\triangle CAD$

$AB = CD$ [Opposite sides of a rectangle are equal]

$BC = DA$

$AC = CA$ Common

$\triangle ACB \cong \triangle CAD$ (SSS Congurency)

Q6. The sides of a rectangle are in the ratio $2 : 3$, and its perimeter is 20 cm . Draw the rectangle.

Solution:

$ABCD$ is a rectangle

Let the side is x

Length of rectangle $= 3x$

Breadth of the rectangle $= 2x$

Given perimeter of rectangle $= 20$ cm

Perimeter of the rectangle $= 2$ (length + breadth)

$20 = 2(3x + 2x)$

$10x = 20$

$x = 2$

Therefore Length of the rectangle $= 3 \times 2 = 6$ cm

Therefore breadth of the rectangle $= 2 \times 2 = 4$ cm

Q7. The sides of a rectangle are in the ratio 4: 5. Find its sides if the perimeter is 90 cm.

Solution:

$ABCD$ is a rectangle

Let the side is x

Length of rectangle $= 5x$

Breadth of the rectangle $= 4x$

Given perimeter of rectangle $= 90$ cm

Perimeter of the rectangle $= 2$ (length + breadth)

$90 = 2(5x + 4x)$

$18x = 90$

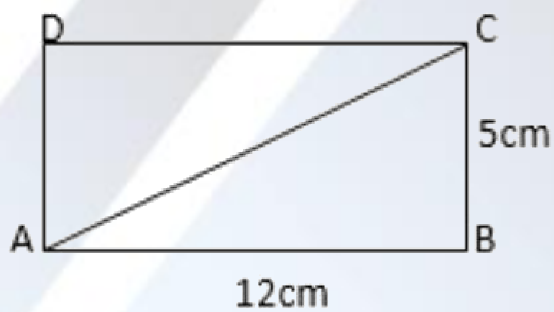
$x = 5$

Therefore, Length of the rectangle $= 5 \times 5 = 25$ cm

Therefore breadth of the rectangle $= 4 \times 5 = 20$ cm

Q8. Find the length of the diagonal of a rectangle whose sides are 12 cm and 5 cm .

Solution:



$ABCD$ is a rectangle

In $\triangle ABC$ using pythagorous theorem,

$AC^2 = AB^2 + BC^2$

$AC^2 = 12^2 + 5^2$

$\mathrm{AC}^2 = 144 + 25$

$AC^2 = 169$

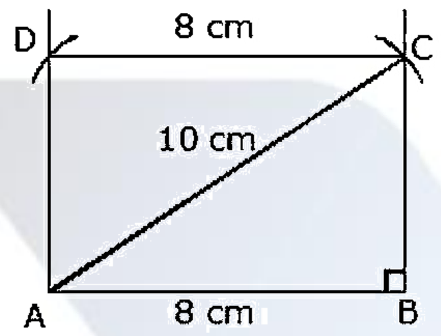
$AC = \sqrt{169}$

$AC = 13$ cm

Therefore length of diagonal is 13 cm .

Q9. Draw a rectangle whose one side measures 8 cm and the length of each of whose diagonals is 10 cm.

Solution:



Steps of construction:

(i) Draw a lien segment AB of length 8 cm

(ii) From point ' A ' draw an arc of length 10 cm .

(iii) From point B draw an angle of 90°, and the arc from point A cuts it at point C.

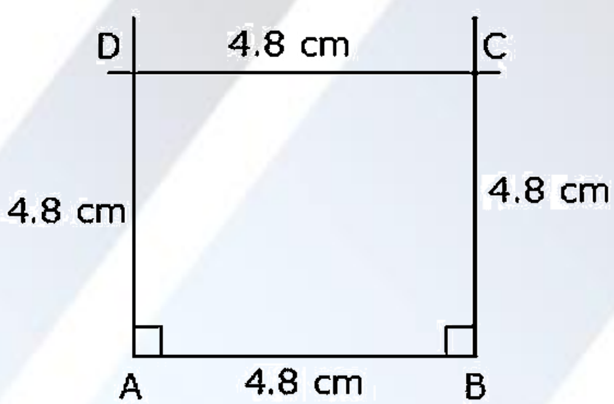
(iv) Join Ac

(v) From point A draw an angle of 90° and point C drawn an arc of length 8 cm to get point D .

(vi) Join CD and AD to get required rectangle.

Q10. Draw a square whose each side measures 4.8 cm.

Solution:



Steps of construction:

(i) Draw a line segment AB of length 4.8 cm .

(ii) From points A and B draw perpendiculars.

(iii) Cut and arc of 4.8 cm from point A and B on the perpendiculars to get point D and C .

(iv) Join DC and AD to get required rectangle.

Q11. Identify all the quadrilaterals that have:

Solution:

(i) Four sides of equal length

Rhombus and square are quadrilaterals that have all four sides of equal length.

(ii) Four right angles

Rectangle and square have all four angles right angles.

Q12. Explain how a square is

(i) a quadrilateral?

(ii) a parallelogram?

(iii) a rhombus?

(iv) a rectangle?

Solution:

(i) a quadrilateral?

A square is a quadrilateral because it has four equal sides.

(ii) a parallelogram?

A square is a parallelogram since it has opposite sides equal and parallel.

(iii) a rhombus?

A square is a rhombus because it has all four sides of equal length.

(iv) a rectangle?

A square is a rectangle because its opposite sides are equal and parallel and each angle is right angle.

Q13. Name the quadrilaterals whose diagonals:

(i) bisect each other

(ii) are perpendicular bisector of each other

(iii) are equal.

Solution:

(i) bisect each other

In a Parallelogram, rectangle, rhombus and square diagonals bisect each other.

(ii) are perpendicular bisector of each other

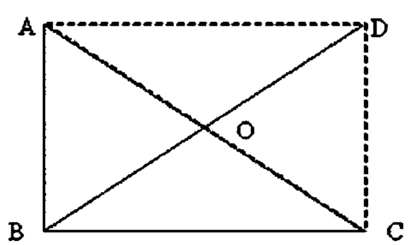
In a Rhombus and square diagonals are perpendicular bisector of each other

(iii) are equal.

In a square and rectangle diagonals are of equal length.

Q14. ABC is a right angled triangle and O is the mid-point of the side opposite to the right angle. Explain why O is equidistant from A, B, and C.

Solution:



ABC is a right angled triangle. O is the mid point of hypotenuse AC, such that $OA = OC$
Draw $CD \| AB$ and join AD, such that $AB = CD$ and $AD = BC$
Now quad $ABCD$ is a rectangle, since each angle is a right angle and opposite sides are equal and parallel.
We know in a rectangle diagonals are of equal length and they bisect each other.
Therefore, $\mathrm{AC} = \mathrm{BD}$
And also, $AO = OC = BO = OD$
Hence, O is equidistant from A, B and C.

Q15. A mason has made a concrete slab. He needs it to be rectangular. In what different ways can he make sure that it is rectangular?

Solution:

a. By measuring each angle, because in a rectangle each angle is a right angle.
b. By measuring opposite sides. Since in a rectangle opposite sides are of equal length.
c. By measuring the lengths of diagonals. Since in a rectangle diagonals are of equal length.